MU120 Unit 3

The Open University

Mathematics and Computing
A first level multidisciplinary course

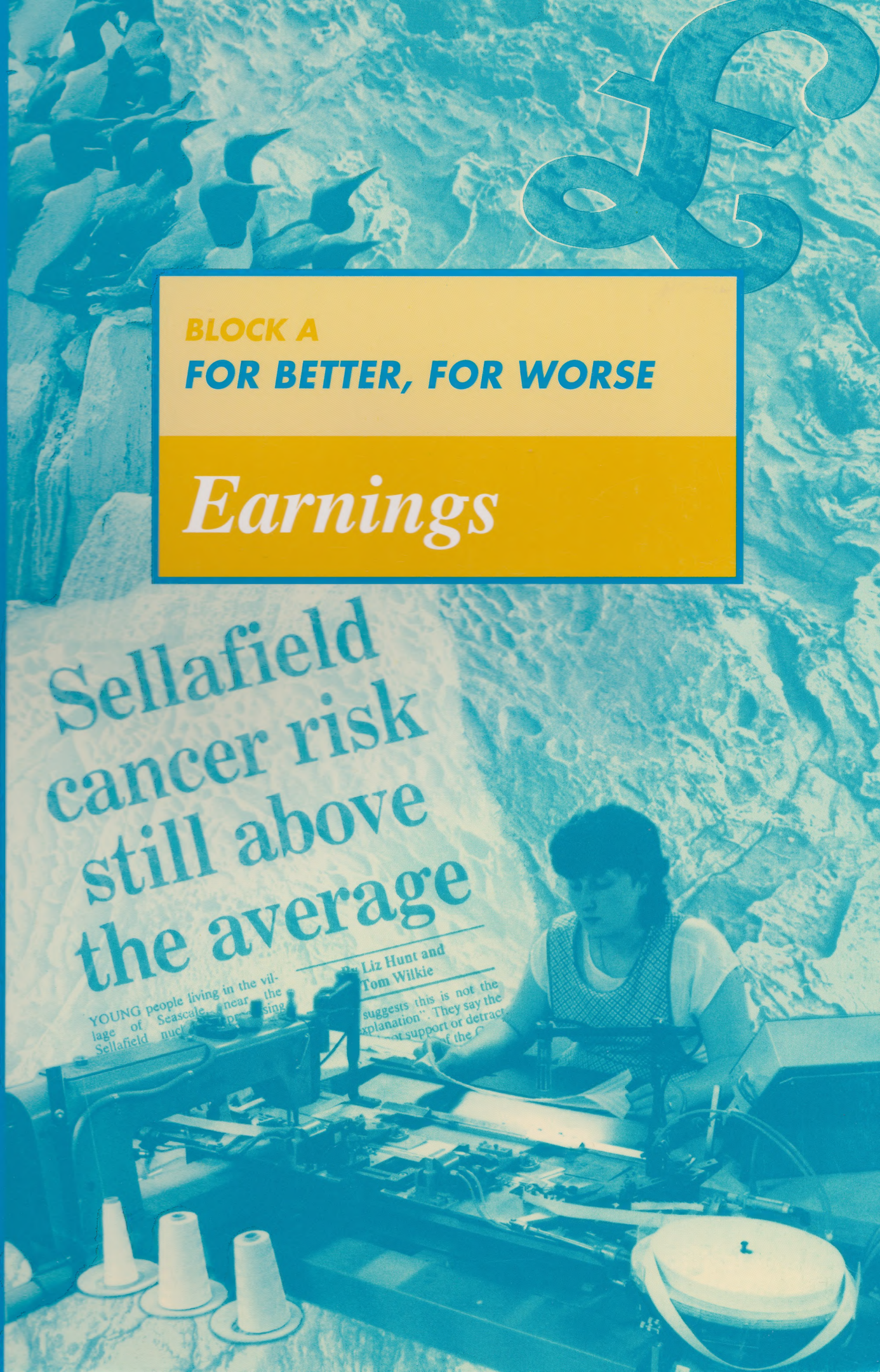
Open Mathematics

UNIT 3

BLOCK A

FOR BETTER, FOR WORSE

Earnings



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Prepared by the course team

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1.5

Contents

Study guide

This unit consists of seven sections. Section 1 involves several activities based on watching a short piece of video. Its purpose is to raise some of the issues that are investigated in the unit and to start you thinking about what might be involved in investigating them. Sections 1 to 6 are closely linked and should be studied in order. However, if you are unable to watch the video at the beginning, this need not prevent you from starting Section 2. Section 6 draws on ideas and techniques discussed in Sections 2 to 5; it may be studied at any time after Sections 2 to 5. Section 7 may be studied at any time and contains an audiotape band. You will need some graph or squared paper for Section 4 and the Appendix which gives details of how to draw accurate boxplots.

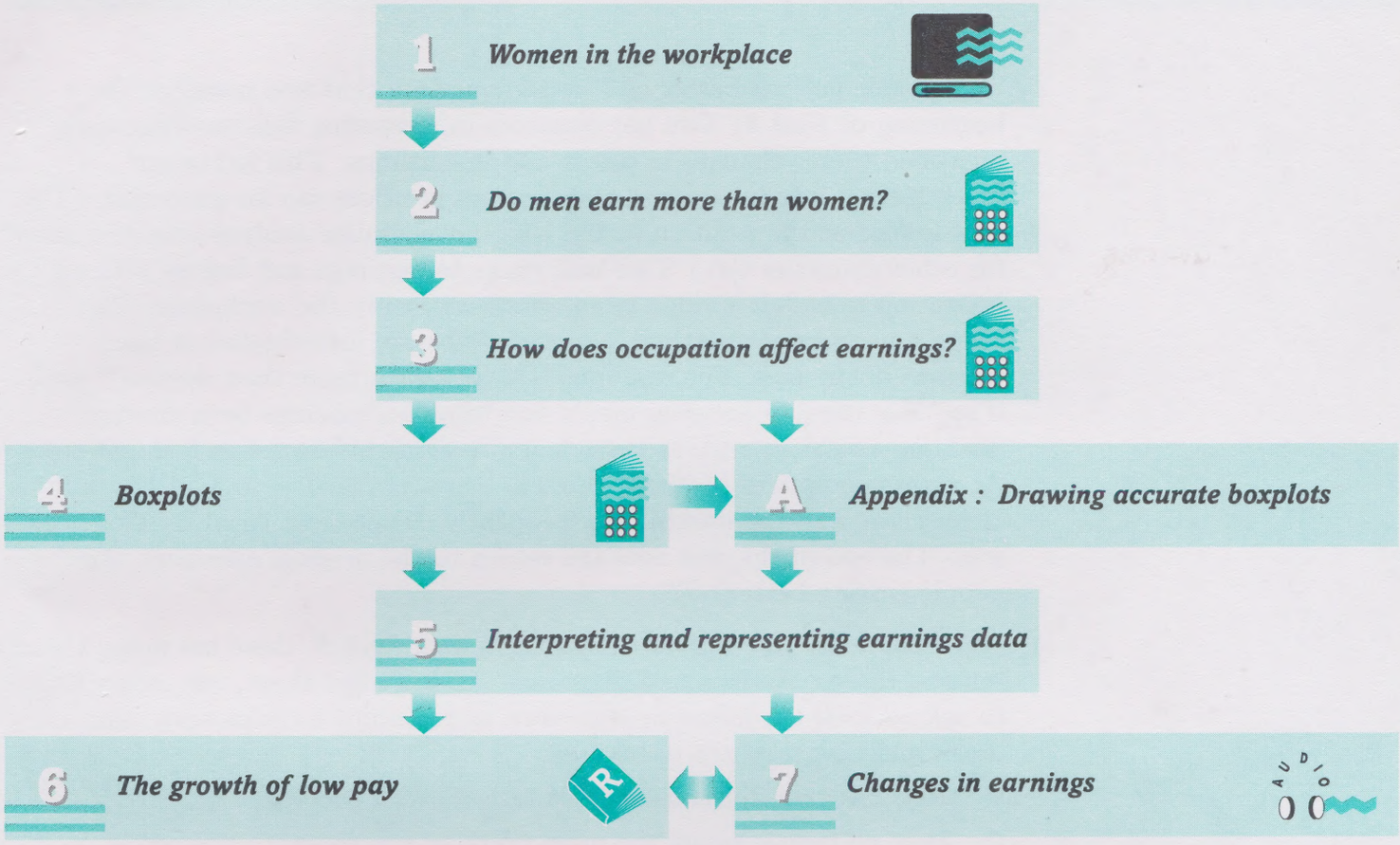
As you work through the unit, you will once again be applying familiar mathematical skills such as calculating percentages. Averages will be discussed further and the idea of an index will again be used—this time to measure changes in earnings. You should also be concentrating on improving your written communication, and developing and applying the more general features of statistical problem solving, as well as representing and interpreting information. There is a series of activities in Section 6 based on a reader article; by working on published material involving mathematical ideas, you should be able to develop further your skill at critical reading.

At the end of *Unit 2*, you were asked to reflect on your progress so far in the course. You should carry out a similar activity at the end of this unit: an activity sheet is provided for this purpose. Remember to make notes of any topics or key ideas that you want to summarize as you come across them. A Handbook activity sheet has been provided for you to enter explanations and descriptions of new terms, but this sheet represents only a bare minimum of what you should record. If you make notes as you work through the unit, then completing the activity at the end of the unit should be a relatively straightforward task.

As you have done for *Units 1* and *2*, devise a plan of action for studying this unit, taking into account the different components that it includes. How you are managing your different priorities and commitments to include your Open University study? Try to include any changes you plan to make to your study for this unit as a response to working through *Units 1* and *2*. It is useful to review practice and progress frequently, as this will enable you to make changes that will be beneficial to the work in hand.

As you work through the unit, keep a record of the targets met and those not met—and then reschedule as necessary.

You also have to complete a tutor-marked assignment question within this study time, so make sure time for this is included in your schedule.



Summary of sections and other course components needed for *Unit 3*

Introduction

'Are people getting better off?' is the question that was posed at the beginning of *Unit 2*. Two key elements in answering this question were identified there: changes in prices and in earnings. This led to an investigation into how prices and changes in prices can be measured. (The unit looked at the position in the UK, but a similar analysis could be used for other countries too.) This unit turns to earnings and begins with an invitation to watch a video band about women in the workplace. This raises a number of questions, some of which are investigated in later sections of the unit. For example, 'Do men earn more than women?' and, if so, 'Has the gap between men's and women's earnings been closing?' Tackling questions such as these is not as straightforward as it might seem. At various points in the text, you will be encouraged to look at the data themselves, and the conclusions frequently drawn from them, in a critical way. The end of the unit sees the return to the original question: 'Are people getting better off?'

One final note: throughout this unit, as with *Unit 2*, there are many visual images, representations and diagrams. As you meet them, you might like to ask yourself the following questions as you come to grips with what they mean and how they are to be used.

- ◇ How clear an illustration of an idea do they provide?
- ◇ How do they support the main point as you see it?

1 Women in the workplace

Aims The main aim of this section is to show how the situation of women who go out to work has changed since the Second World War, and to discuss how statements about women's and men's earnings might be investigated. ◇

Your work in this section is based on the video band 'Women in the workplace'. At various stages, you will be asked to stop the video and answer some questions about what you have just seen and heard. The four sequences move progressively in time from 1939 to the 1990s. Each time, before restarting the video, return to this text to read the comments.

Now watch the first section of band 2 of Videotape 1.

Extract 1 Bella the welder

You may recall you saw a short sequence involving Bella as part of the introductory videotape material from *Unit 1*. The focus here is slightly different.

At the end of this video sequence, you were asked the following questions.

- ◇ What information have you remembered?
- ◇ What question would you most like to ask Bella?

The purpose of these questions was to encourage you to be precise about what you noticed and to think about what further information you would like to have. What sorts of question did watching and listening to Bella raise for you?

- ◇ Did you want to know more about working conditions for women in the war?
- ◇ Did you wonder how changes in attitudes to women and work came about in the years following the war?
- ◇ Were changes slow and steady or were there periods of rapid change resulting from particular events?

In the video commentary, a distinction is made between *quantitative* data (information based on measurements, usually given as numbers) and *qualitative* data (information in non-numerical form). Quantitative data are usually more straightforward to record and process. But qualitative data can be altogether more subtle and difficult to interpret. Some information is conveyed both in numbers and words—for example, Bella's earnings and hours worked or the attitudes of her male colleagues to working with women—whereas other information is conveyed in her voice and expression. Did you notice how the tone of Bella's voice and her expression altered when she described the completed ship and called it 'a thing of beauty, a wonderful thing'? There is a wealth of information here about her feeling of pride and awe in her achievement and that of her colleagues—information which could not be gained from a written transcript of her words alone.

Traditionally, statistics has focused on information which can be quantified, but that is only a small subset of the wide variety of information that we process when functioning as thinking and feeling human beings. Decisions are regularly made not just on facts that can be written down, but also on how a person looks, speaks, acts, and so on. Your reaction to what Bella had to say was almost certainly influenced by the way she said it.

Now watch the second section of band 2 of Videotape 1.

Extract 2 The Equal Pay Act

At the end of the second extract, you were asked the following questions.

- ◇ How did the Equal Pay Act affect the women interviewed?
- ◇ What do you conclude about the immediate effects on women of the Equal Pay Act?

One of the messages of this sequence was that the aims of well-intentioned legislation can sometimes be thwarted. According to the part-time farm workers, Joyce Bottomley and Rita MacNeil, their employer did this by reducing their hours of work to thirty per week, the threshold *above which* their employer was required by law to pay full-time rates—the rates earned by the men. And, in order to prevent the cutter, Cissie O'Keefe, claiming equal pay, her work was changed so that she would no longer be doing the same job as a man.

A word of caution is appropriate here. Be careful not to make over-generalizations from individual cases. Remember that these sequences were selected by the programme makers to represent sympathetically the views of these female workers. You do not know how typical these case histories were, and no attempt was made to present the employers' or unions' position.

Now watch the third section of band 2 of Videotape 1.

Extract 3 Equal pay for work of equal value

Following pressure from the European Economic Community (now called the European Union), the Equal Pay Amendment Regulations were enacted in the UK and came into effect in January 1984. In the third extract, you heard about the claims of several women that their work was of equal value to that of men doing different jobs. You were asked the following questions.

- ◇ On what basis can a woman claim that her work is of equal value to a man's?
- ◇ Do you think the claims of the women interviewed were justified?

The Equal Pay Amendment Regulations gave women the right to claim 'equal pay for work of equal value to a man's in terms of effort, skill and training'. It is difficult to decide whether two jobs are of equal value. Nevertheless, the Amendment Regulations do provide means for women to challenge the fairness of a system in which many largely female

occupations are low-paid. Recall the machinists in the clothing factory in the second extract: they received lower wages than the male cutters principally because their occupation was seen as a 'female' one, not because they were less skilled than the men. Their manager admitted that this was probably not fair. When the extract was recorded, before the Amendment Regulations came into effect, they had no right in law to claim equal pay to the men, since their work was 'different' from the men's. Such workers were in a stronger position from January 1984 onwards. However, in practice, most workers have found it extremely difficult to establish that two different jobs are of equal value.

Now watch the fourth and final section of band 2 of Videotape 1.

Extract 4 A trade union view

In the final section of the video band, you saw part of a speech that Audrey Wise gave at the TUC conference in 1991. You were asked to consider the following two questions.

- ◇ What were some obstacles to achieving equality for women?
- ◇ Why do you think she stressed the *hourly* rate when comparing men's and women's earnings?

The two obstacles to equality mentioned by Audrey Wise were the attitudes of employers and the attitudes of men. There are many possible points that you may have thought of when answering the second question. Quite probably you have read or heard that a far greater proportion of women than men work part-time. Since this is the case, it would not be fair to compare the weekly wages of female part-time workers with the weekly wages of men working full-time. It would be fairer to compare the amounts that men and women receive for similar amounts of work. This is a possible explanation for the speaker stressing 'the hourly rate' when comparing men's and women's earnings.

Activity 1 The earnings gap

You have seen various people—journalists, a politician, a trade unionist, an employer, several employees—talking about how women's earnings have compared with men's in the past in the UK and how the situation has changed.

Write down three claims that were made in the video band, either in words alone or with the use of diagrams, that might be investigated with the aid of suitable data.

Think about how you might go about investigating the accuracy of these statements. What data might be needed in each case? For example, would you need data for earnings or for average earnings or for changes in earnings? You might find it helpful to watch the video band again in its entirety, but this time just stop the tape at appropriate points while you write down your ideas, rather than necessarily at the end of each extract.

Statements were made about the relative earnings of men and women on the video band. You have heard how, during the Second World War, male and female welders in the shipyards were paid different rates for doing the same work. Nowadays, this would be against the law. The Equal Pay Act of 1970, fully implemented in 1975, provides for 'equal pay for men and women doing work of a broadly similar nature'. And recall that the Equal Pay Amendment Regulations (1984) gave women the right to claim 'equal pay for work of equal value to a man's in terms of effort, skill and training'.

Various statements were then made about how the gap between women's and men's pay has changed over the years. Look briefly at just three of them. First, Judith Hann described how the average hourly earnings of women changed compared with men's during the 1970s. She used the diagram in Figure 1 to illustrate her statements.

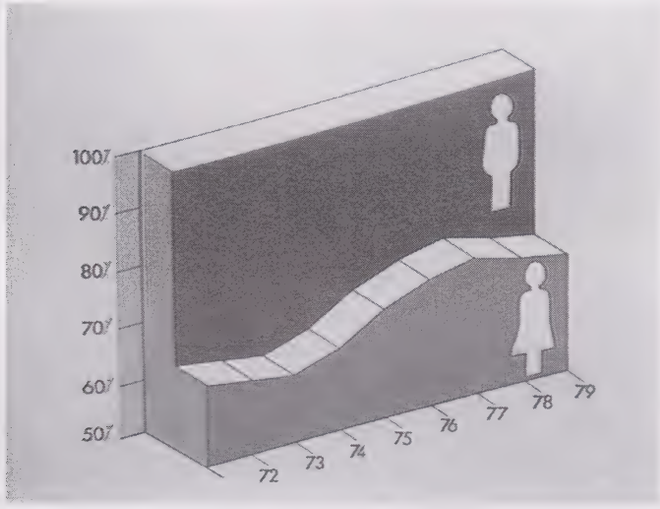


Figure 1 Average hourly earnings of women as a percentage of men's

Source: 'Women at Work', *The Risk Business*, 14 November 1979

The diagram in Figure 2 was shown in a 1984 news item about women's earnings.

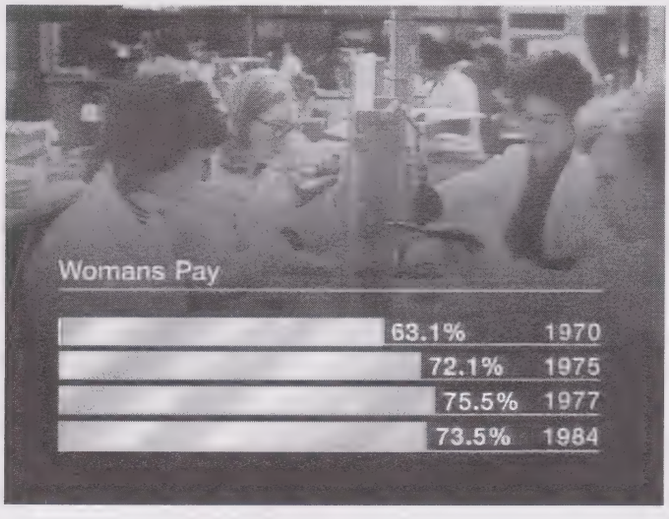


Figure 2 Women's earnings as a percentage of men's

Source: 'Women at Work', *Newsnight*, 30 October 1984

It would appear that the percentages for 1977 in these two diagrams do not agree. So did one of the programmes have incorrect figures?

Before accepting any statement at face value, it is worth asking several questions.

- ◇ On what data is the statement based?
- ◇ Could the data be interpreted in a different way?
- ◇ Have the data been selected to support or stress a particular viewpoint? Have other data, which might lessen the strength of the case, been ignored?
- ◇ If two statements are contradictory, are they based on the same data?

Looking first at Figure 1, you can see that the percentages have been calculated from 'average hourly earnings'. Having studied *Unit 2*, you are now in a position to ask 'Does this refer to *mean* hourly earnings?' Or might it refer to *median* hourly earnings? You cannot tell. And does it matter whether the mean or median has been used? This last question is investigated further in Section 2 of this unit.

In Figure 2, there is no mention at all of an average, but presumably some sort of 'average' earnings data have been used. But they could be data for hourly earnings or weekly earnings or earnings for some other length of time. Again, does it matter? This is another point worth investigating.

Finally, have the earnings of *all* workers, full-time and part-time, been included in the data used? Without knowing the answers to these questions, it is difficult to accept either set of figures at face value or to sort out the apparent disagreement. What exactly do the figures tell us?

At the end of the video band, you heard Audrey Wise, the speaker at the 1991 TUC conference, say 'in 1989, women non-manual workers still earned only 62% of the hourly rates of their equivalent males'.

- ▶ How does this figure compare with the earlier figures given? Is it valid to compare it directly with the earlier ones?

- ▶ What information would be needed to investigate claims such as these? Think about these questions before you read on.

Comparison is probably not valid, assuming that the earlier percentages were based on the earnings of *all* workers and not just on those of 'non-manual' workers. In all these cases, data on the *level* of earnings of men and women would be required for different years. You would need hourly earnings data to check the figures in Figure 1. To check Figure 2, you might also need weekly earnings data. To investigate Audrey Wise's claim, you would need data on hourly earnings of 'non-manual' male and female workers.

In fact, data of this type are available in an annual publication called the *New Earnings Survey*. These data could be used to investigate the percentages quoted in all the examples above. However, rather than investigate the situation as it existed in the 1970s and 1980s, you will be

looking at women's and men's earnings in the early 1990s. In Sections 2 to 5, data from the 1993 *New Earnings Survey* (Volumes A and D) will be used to investigate the question 'Do men earn more than women?'

The New Earnings Survey

The *New Earnings Survey* has been carried out by the Department of Employment (now the Department for Education and Employment) each April since 1970. Its main purpose is to provide information for the British government about earnings within the UK. The survey provides information on approximately 170 000 men and women who are all members of the Inland Revenue pay-as-you-earn (PAYE) scheme; it does not cover the self-employed, nor, of course, the unemployed.

Each member of the PAYE scheme has a National Insurance number such as YR 04 73 49 D. In any year, all employees whose National Insurance numbers end with a particular pair of digits are included in the survey. (The final *letter* is ignored.) The employer of each selected employee is contacted by post and is required by law to provide certain information. The data collected include the following information concerning the employee.

- ◇ Gender and age.
- ◇ Location of workplace.
- ◇ Occupation.
- ◇ Total gross weekly earnings during a specified week. (For monthly-paid workers, equivalent weekly earnings are calculated from their month's salary.)
- ◇ Information about normal basic hours, overtime earnings and hours, and bonus payments.

The information collected is analysed by government statisticians and the results are published in a number of volumes.

Outcomes

After studying this section, you should be able to:

- ◇ specify some types of data needed to investigate claims and comparisons about women's and men's earnings.

2 Do men earn more than women?

Aims The main aim of this section is to show how the mean and the median may be used to make meaningful comparisons between the earnings of men and women. ◇

In Sections 2 to 5 of this unit, you are going to concentrate on the question 'Do men earn more than women?' There will also be more opportunity for you to work with the statistical problem-solving processes mentioned in *Unit 2*. In this section, the choice of appropriate data on the earnings of men and women and how to compare them is discussed.

2.1 Data—comparing like with like

Differences in earnings between individuals can be accounted for by a great variety of factors apart from gender.

▶ Spend a minute or two thinking about what these factors might be and write down your ideas. (Try to write down at least two ideas before you read on.)

You may have suggested factors such as education, training, social background, age, occupation and number of hours worked. Some people earn more than others for doing similar work. Some jobs are rewarded more than others: danger, level of responsibility and skill required are all factors affecting pay. Personal qualities such as aptitude, ability and temperament also have some effect. You may have thought of others, such as geographical location, nepotism, status, historical precedent: there is a multitude of factors.

Some factors may be more important than gender in determining levels of earnings, some less important. A number of these factors may be interrelated with gender, producing even more complexity. For example:

- ◇ more women than men work part-time, so any difference between the earnings of men and women may be due to this;
- ◇ the proportion of female workers who are on adult pay rates is smaller than the proportion of male workers on adult rates, so this too could affect earnings differences;
- ◇ some jobs are done predominantly by women, others mainly by men, and earnings vary historically from job to job, so this is another possible reason for any differences in earnings.

Activity 2 *Separating the women from the girls*

It was stated above that the proportion of female workers being paid adult rates is smaller than the corresponding proportion of male workers. Suggest a reason for this.

Comments on Activities begin on page 75.

Since the various factors which affect earnings are so numerous and almost all are interrelated, it is impossible to disentangle them. However, if the earnings of men and women who are similar with respect to some of these factors are compared, then some possible sources of distortion in the results obtained will be eliminated. For example, male chefs working full-time should probably not be compared with female doctors working part-time. It would be better to compare men and women working full-time, and men and women in the same occupation. As far as possible, it is good to *compare like with like.* But, since the investigation depends on published data, which factors *can* be taken into account is to some extent limited. This is an important point to bear in mind when you are planning and carrying out your own statistical investigation.

In 1993, more women than men worked part-time and a smaller proportion of female workers was on adult rates than was the case for men. To attempt to eliminate the effect that these factors have on the relative pay of men and women, the investigation will only concern men and women working full-time and being paid adult rates; and it only applies to employees whose pay was not affected by absence in the pay-period for which the data were collected.

All the data in Sections 2 to 5 relate to men and women working full-time on adult pay rates and whose pay was unaffected by absence.

Note that identifying the salient variables and deciding on appropriate measures of them is an important aspect of any statistical (and often scientific) investigation.

Before considering the effect of occupation on pay, first consider any difference between the overall earnings of men and women. Table 1 gives the mean gross weekly earnings of adult men and women in full-time employment in the United Kingdom in 1993.

Table 1 Mean gross weekly earnings of adult men and women (to the nearest pound)

	Women	Men
Mean	253	354

Source: *New Earnings Survey*, 1993, Table A23.2

Having obtained some data, the next step is to decide how (and what mathematical strategies to use) to compare the earnings of men and women. From Table 1, it is clear that the mean gross weekly earnings for men is greater than the corresponding figure for women. Two methods of comparing men's and women's mean earnings probably spring to mind: subtracting one from the other to find the (numerical) difference, and dividing one by the other to find the ratio.

Activity 3 Comparing men's and women's earnings

Compare the mean gross weekly earnings of men and women by a method of your choosing. Make a brief note to explain why you chose that method.

First, consider the numerical difference: this is £(354 − 253) = £101. So the mean gross weekly earnings of adult men is £101 more than the mean gross weekly earnings for women.

▶ Is this a useful way of comparing the earnings of men and women?

Suppose that the mean weekly earnings of men and women had been £154 and £53, respectively; in this case, the difference would also have been £101, as it would have been if the weekly earnings had been £1354 and £1253. However, a difference of £101 would be regarded as of much greater importance in the first case than in the second. So it would be better to know something about the *relative* size of the difference and not just the absolute difference. Recall from Chapter 1 of the *Calculator Book*, the discussion of price increases in a pint of milk and a car between 1984 and 1994. Finding the difference there was not too useful a comparison, and the same may be true here. But to talk about the 'gap' between men's and women's earnings may unhelpfully suggest numerical difference as the appropriate measure.

Now consider their ratio. In *Unit 2*, ratios were used to compare prices. One of the benefits of using ratios is that whatever the unit of measurement—pounds, pence, francs, dollars—the ratio remains the same. So whatever the unit of measurement used to record earnings, the ratio of women's earnings to men's earnings would be the same. Recall the loaf of bread measure in Section 1 of *Unit 2*: you were able to compare ratios for 1594 and today even though the units of measurement were different—old pence in 1594 and new pence today. Moreover, you were able to do this even though both prices and earnings had changed enormously over the four-hundred-year period. The ratios did not depend on the absolute size of the quantities being compared, only on their relative size. So even ratios calculated at different times can still be meaningfully compared. Using ratios would therefore make it possible to extend the investigation to make international comparisons or to make comparisons over time.

Earnings *ratios* will therefore be used. And since the available data are the *mean* gross weekly earnings for men and for women, take the ratio of these means as the earnings ratio. The technical term for this is the *earnings ratio at the mean*, and it is defined as follows.

The *earnings ratio at the mean* is: $\dfrac{\text{mean earnings of women}}{\text{mean earnings of men}}$

Note that the earnings ratio at the mean is taken to be the mean women's earnings divided by the mean men's earnings, rather than the other way round. This is an established convention.

▶ Why do you think this convention is used, and what are some of its effects?

Example 1 *Calculating an earnings ratio at the mean*

For the data in Table 1, the earnings ratio at the mean is

$$\frac{253}{354} \simeq 0.7147.$$

Earnings ratios are often expressed as percentages. Thus, the mean gross weekly earnings of adult women in full-time employment in 1993 was approximately 71% of the mean gross weekly earnings of adult men in full-time employment.

One arithmetic effect of the convention mentioned above, in a context where men usually earn more than women, is that the earnings ratio at the mean will therefore usually be less than one (or a percentage less than 100). The nearer the earnings ratio at the mean is to 100%, the nearer 'average' earnings of women are to those of men. In this unit, all earnings ratios will be expressed as percentages rounded to the nearest whole number.

Table 1 gives the mean gross (that is, before any deductions, such as tax, pension, national insurance, are removed) weekly earnings of adult men and women in full-time employment. By selecting these data rather than corresponding data for *all* men and *all* women, some factors which affect the earnings ratio at the mean have been taken into account. These are as follows: proportionately more women than men work part-time, and proportionately fewer female workers than male workers are on adult rates. As previously explained, there are many other factors that affect earnings: total hours worked, amount of overtime and occupation.

▶ Do any of these factors have an effect on the relative earnings of men and women?

If they do, then, in order to make a fair comparison, they should be taken into account. The next questions therefore need to be: how can you find out what effect they have? And what can be done to take any effects into account? Notice here 'taking into account' means excluding them from the comparison. The principle of comparing 'like with like' leads to a comparison based on a much smaller number of people.

Look first at hours worked and overtime. Of course, both these factors are related to occupation, and since some occupations attract more men than women and vice versa, you might expect there to be differences between the hours worked and overtime of men and women.

Table 2 Mean weekly hours worked by adult men and women in full-time employment in the UK in 1993 (to one decimal place)

	Women	Men
Normal basic	36.5	38.2
Overtime	0.8	3.1
Total	37.4	41.3

Source: *New Earnings Survey*, 1993

Activity 4 Hours and overtime

(a) On average, how many hours did women work per week in 1993? Was this more or less than the average number of hours worked by men?

(b) On average, did men or women do more overtime per week in 1993, and by how much?

(c) What do you think the effect would be of excluding overtime pay from the mean gross weekly earnings used to calculate the earnings ratio at the mean? Do you think this earnings ratio would increase or decrease if overtime were excluded?

(d) Men and women work a different number of hours per week on average. Can you suggest a way of eliminating any effect due to this?

In this context, the everyday phrase 'on average' means using the mean.

Table 3 below gives the mean gross weekly earnings (in pounds), excluding overtime, and the mean gross hourly earnings (in pence), excluding overtime effects, of men and women in the UK in 1993.

Table 3 Two earnings means

	Women	Men
Mean gross weekly earnings excluding overtime (£)	247	330
Mean gross hourly earnings excluding overtime (pence)	668	847

Another possibility would be to divide the figure for the mean gross weekly earnings from Table 3 by the mean basic hours worked (from Table 2).

$$\frac{\text{mean gross weekly earnings for men}}{\text{mean basic hours worked}} = \frac{£330}{38.2\,\text{hrs}} \simeq £8.64 \text{ per hour.}$$

But the figure in Table 3 is 847p = £8.47. You may find it surprising that the two are different. If so, then you may find the illustration of this phenomenon given in the following example helpful.

There is more than one way of defining the mean of gross hourly earnings. The way the figures given in Table 3 were calculated was by adding up the hourly rates of all the people in the batch and dividing by the number of people in the batch.

Example 2 Mean earnings and hours worked

Ivan earns £175 for a basic working week of 35 hours; that is, he earns £5 an hour. Keith earns £405 for a 45-hour week; that is, he earns £9 an hour. Calculate both the mean hourly earnings and the ratio:
mean weekly earnings/mean hours worked for Ivan and Keith combined.

Their mean weekly earnings figure is

$$£(175 + 405)/2 = £290,$$

and their mean number of hours worked is

$$(35 + 45)/2 = 40.$$

So

$$\frac{\text{mean weekly earnings}}{\text{mean hours worked}} = \frac{£290}{40\,\text{hrs}} = £7.25 \text{ per hour.}$$

The mean hourly earnings is

$$£(5 + 9)/2 = £7.$$

Activity 5 Calculating earnings ratios

In Example 1, the earnings ratio at the mean based on gross weekly earnings including overtime was found to be 71%. Use the data in Table 3 to:

(a) calculate the earnings ratio at the mean based on gross weekly earnings excluding overtime;

(b) calculate the earnings ratio at the mean based on gross *hourly* earnings excluding overtime;

(c) describe the effect on the earnings ratio at the mean of excluding overtime and using data for hourly earnings instead of weekly earnings.

Removing overtime pay from gross weekly earnings increases the earnings ratio at the mean from 71% to 75%. Comparing hourly earnings instead of weekly earnings increases the earnings ratio at the mean further to 79%. So the longer average working week and extra overtime worked by men account for at least part of the difference between the weekly earnings of men and women. Therefore, to find out whether or not groups of men and women receive equal pay for a similar amount of work, use gross hourly earnings excluding overtime (when available) for a fairer comparison. Notice this does not take into account the possibility that men may have a greater opportunity to work a longer week or undertake overtime: the raw fact at the end of the working week is still more money for men. This could reflect a different form of unequal opportunity.

2.2 Has the 'gap' between men's and women's earnings been closing?

To investigate this question, data on earnings are needed for a number of different years. Table 4 shows the mean gross hourly earnings excluding overtime for adult employees for various years between 1979 and 1993.

Table 4 Mean gross hourly earnings excluding overtime (in pence)

Year	Women	Men
1979	166	227
1981	241	323
1983	288	388
1985	330	445
1986	358	482
1987	384	521
1988	427	568
1989	476	623
1990	525	682
1991	587	750
1992	635	803
1993	668	847

Source: *New Earnings Survey*, various years

Activity 6 Changes in the earnings ratio over time

(a) Calculate the earnings ratio at the mean for each year in Table 4.

(b) How has the earnings ratio changed since 1979?

(c) On the evidence of your calculations, would you say that gender inequalities in earnings have widened, narrowed or stayed the same between 1979 and 1993?

2.3 Averages—the mean or the median?

So far in investigating men's and women's earnings you used the arithmetic mean when comparing levels of earnings. But there are other averages—the median, in particular. Would the results of the investigation have been the same using *median* earnings of men and women?

The mean earnings of any group of people may be thought of as the 'average' of the earnings of all the people in that group. The median earnings may be thought of as the earnings of the 'average person' in the group—roughly speaking, 50% earn more than the 'average' person and 50% earn less. (It would be more accurate to say that 50% earn the same or more than the 'average' person and 50% earn the same or less.) So mean earnings and median earnings are different ways of measuring the 'middle' level of earnings of the group.

Look back to your notes on the *Unit 2* Handbook activity and decide whether you want to add your entries for *mean* and *median*.

Table 5 gives values of the median and mean gross weekly earnings including and excluding overtime, and the median and mean gross hourly earnings excluding overtime for adult men and women in full-time employment in 1993.

Table 5 Some 1993 earnings data

	Median		Mean	
	Women	Men	Women	Men
Gross weekly earnings incl. overtime (in £)	222	305	253	354
Gross weekly earnings excl. overtime (in £)	214	276	247	330
Gross hourly earnings excl. overtime (in p)	580	713	668	847

Source: *New Earnings Survey*, 1993

The *earnings ratio at the median* is defined in a similar way to the earnings ratio at the mean: it is calculated by dividing the median earnings for women by the median earnings for men.

The *earnings ratio at the median* is: $\dfrac{\text{median earnings of women}}{\text{median earnings of men}}$

Activity 7 The earnings ratio at the median

(a) Calculate the earnings ratio at the median using the data in Table 5 for each of the following: gross weekly earnings including overtime, gross weekly earnings excluding overtime, gross hourly earnings excluding overtime.

(b) Compare the three earnings ratios at the median that you calculated in (a) with the corresponding earnings ratios at the mean (which were calculated in Example 1 and Activity 5). This latter set of values is 71%, 75% and 79%, respectively. What do you notice?

The comments on this activity (page 76) observe that in each case the earnings ratio at the median is greater than the earnings ratio at the mean. Remember the nearer any earnings ratio is to 100%, the closer the earnings of women are to those of men. So the relative 'gap' between the earnings of the 'average' man and the 'average' woman (that is, at the median) is less than that between the 'average' earnings of all men and the 'average' earnings of all women (that is, at the mean).

Looking again at Table 5, you can see that, for both men and women, the median earnings figure is less than the mean earnings figure. In fact, for earnings data, it is generally true that the median is smaller than the mean. Why should this be so?

Here is an example of a typical *distribution* of earnings; that is, how earnings vary between employees. Imagine a small manufacturing company, for instance. The earnings of the majority of the employees will probably not be very different from one another: maybe some will earn as much as twice the amount that others do, but not much more. However, there will almost certainly be one or two senior managers who earn very much more. This hypothetical distribution of earnings is illustrated in Figure 3.

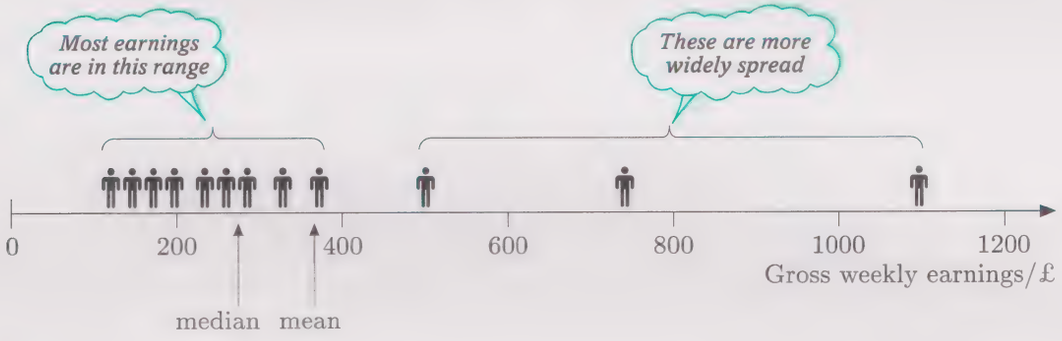


Figure 3 The distribution of earnings in a small (imaginary) company

So the earnings of the majority of employees will be fairly closely grouped, but there will be a few who earn much more. This is the case for earnings in general. The earnings of the majority of employees do not differ from each other a great deal, but a minority of people earn very much larger amounts. This distribution of earnings is the primary reason for the phenomenon that median earnings are generally lower than mean earnings. The following activity should help you to see how this occurs.

For this activity, you need to be able to enter and edit data in the lists or statistical registers of your calculator, and to use your calculator to find the mean and the median of a list of data.

If you are not sure how to do this, then look again at Section 2.1 of Chapter 2 of the Calculator Book.

Activity 8 *A calculator investigation*

(a) Troublefree Computers has five employees, including the manager. Their weekly earnings in pounds are 200, 250, 300, 350, 400. Find the mean earnings and the median earnings of the employees.

(b) Suppose that one employee, the manager, is given a rise from £400 to £500. Calculate the mean earnings and the median earnings now. Which is the larger figure?

(c) Now vary the amount received by the manager, keeping the earnings of the other employees unchanged. Find the mean earnings and the median earnings for each of a number of different values of the manager's new salary, each larger than £400. What do you notice?

In Activity 8, you saw that as the manager's pay is increased, the median is unaffected, but the mean earnings increases. This is illustrated in Figures 4 to 6.

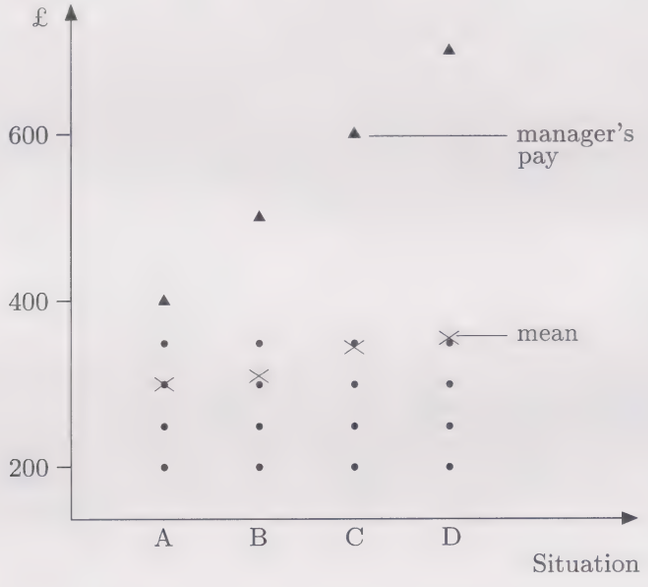


Figure 4

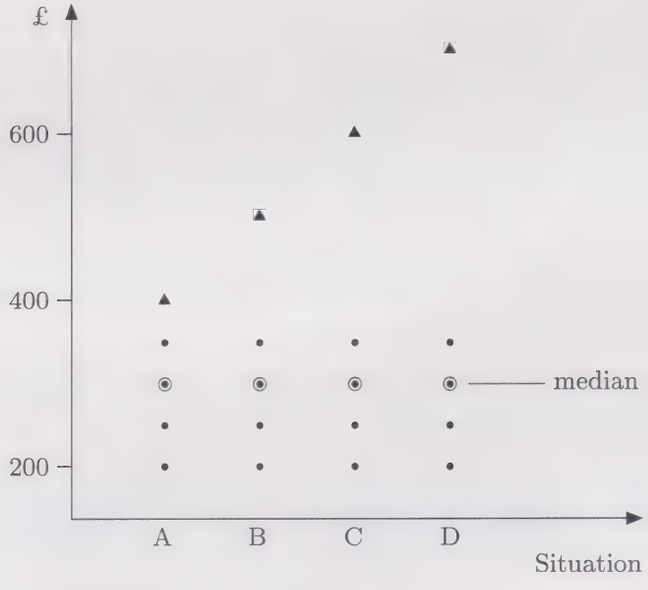


Figure 5

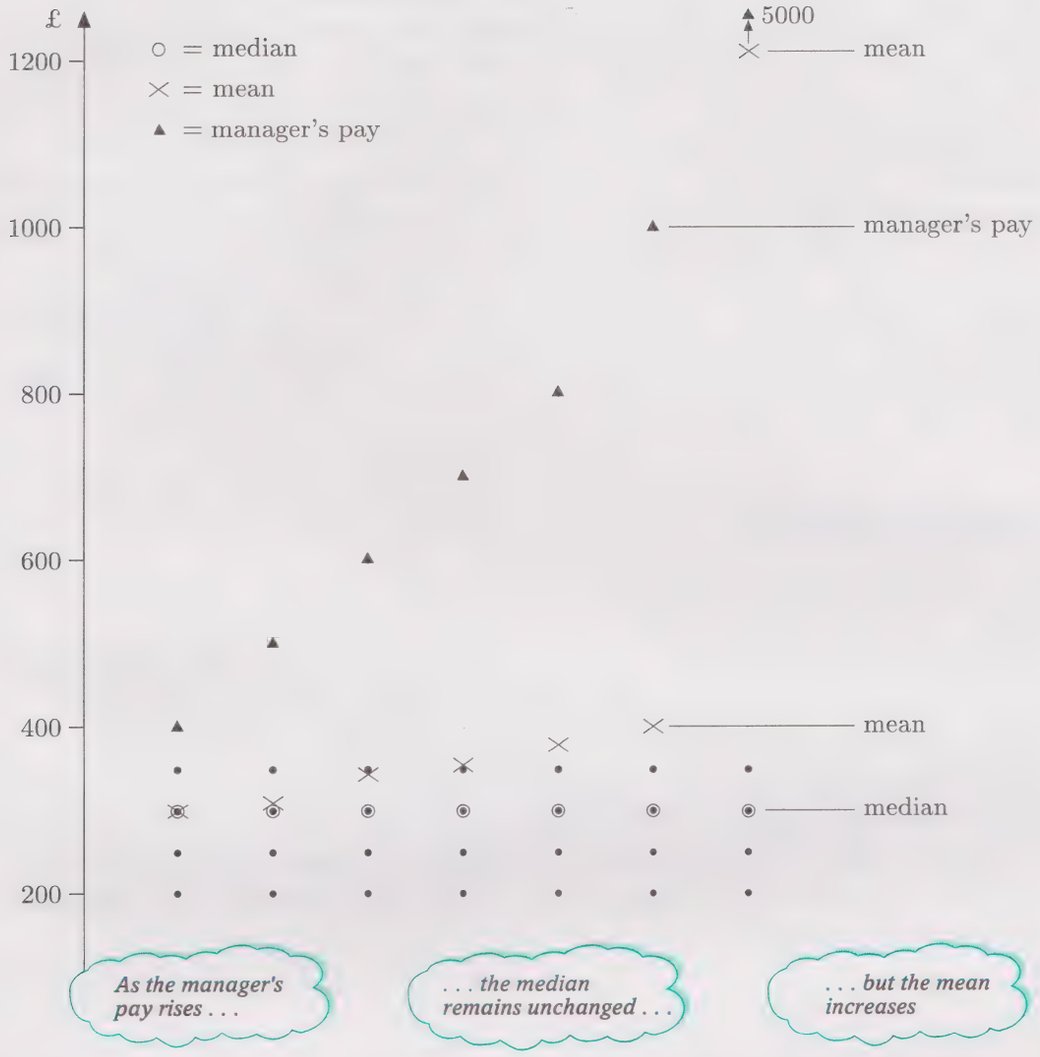


Figure 6 The mean and the median as the manager's earnings increase

The median is unaffected, because the middle value remains the same however large the manager's pay becomes. The mean, however, is the total earnings of the five employees divided by five; so, as the manager's pay increases, the total earnings of the five employees increases and hence the mean rises. Increasing the manager's pay has the effect of dragging up the mean.

Since, in general, higher earnings are much more widely spread than lower earnings, this phenomenon of the mean being greater than the median is to be expected when examining earnings data. Further, the more widely spread the higher earnings are, compared with the lower earnings, the greater the difference between the mean and the median will be. For the earnings data in Table 5, the mean is greater than the median for both men and women. This reflects the fact that higher earnings are more widely spread than lower earnings for both men and women. Also, in Activity 7, you found that the earnings ratio at the median is greater than the earnings ratio at the mean in each case. This is due to higher earnings being more widely spread compared with lower earnings for men than for women.

Since mean earnings are generally higher than median earnings, a trade union official might use median earnings to support an argument that average pay is low. On the other hand, the employer might use the mean to argue the opposite. Thus, decisions about which measure to use have implications for how the data are interpreted. Since the relatively large earnings of a minority can have such a marked effect on mean earnings, whereas the median is unaffected by a few extremely large values, there is a case for regarding the median as more representative of earnings in general than the mean. In practice, the median is more commonly used in official sources than the mean when dealing with earnings data. For the rest of the investigation into the earnings of men and women here, we use the median rather than the mean.

The pay parade

The following image appeared in a journal article in 1994. Imagine a parade of all the workers in the UK in which everyone's height is proportional to their weekly earnings: so a person earning an average (that is, mean) wage is of mean height. The shortest, that is the lowest-paid, pass by first, the tallest last. Suppose the parade takes an hour with everyone moving at the same speed. For the first forty-five minutes all you see are very short people—ten million people less than four feet tall. Only in the last twelve minutes do you see people of average height, followed by a few giants: government ministers ten metres tall and heads of companies as tall as a skyscraper.

Source: *The New Review*, No. 24

Activity 9 The pay parade

Suppose that the parade begins at 10 am and ends at 11 am.

(a) At what time would a person of median earnings pass by?

(b) According to the description above, at what time does a person earning the mean wage pass by? What percentage of people earn less than the mean wage?

(c) Think about the image itself: what were some of your reactions to it? Were there any aspects that confused you or where you felt you were being misled?

So far, in investigating the relative earnings of men and women, several factors that affect earnings have been taken into account. Since more women than men work part-time, the investigation was restricted to full-time workers. Since proportionately fewer female workers than male workers are paid as adults, only workers on adult pay rates were considered. Since men work more overtime on average than women, overtime was excluded. Since the normal basic working week is slightly longer on average for men than for women, the average hourly earnings of men and women were compared instead of the average weekly earnings. This is summarized in Table 6.

Table 6 Adjustments made in order to compare 'like with like'

Perceived problem	Proposed solution
More women than men work part-time.	Look only at full-time workers.
Proportionately fewer women workers are on adult pay rates.	Look only at workers on adult rates.
Men work more overtime.	Exclude overtime.
Men work a longer basic working week.	Compare hourly earnings.

But remember too what (and who) has been ignored in order to satisfy this general principle of comparing 'like with like'.

Even after taking all these factors into account, the earnings ratios at the mean and at the median for 1993 were about 80% (79% at the mean and 81% at the median). So it appears that, on average, adult women working full-time are paid about 80% of the amount paid to men for an hour's work. Does this mean that women and men are not receiving equal pay for equal work? Or are there other important factors in determining pay that have not yet been taken into account?

Perhaps the most important factor that has not yet been considered is actual occupation; this is investigated in the next section.

Outcomes

After studying this section, you should be able to:

- ◇ comment critically on given data and their appropriateness before doing any calculations (Activity 2);
- ◇ interpret data accurately from tables (Activities 3 and 4);
- ◇ explain the meaning of the term 'earnings ratio' and to calculate its value at the mean and at the median given relevant data (Activities 5, 6 and 7);
- ◇ draw general conclusions from earnings ratio calculations given appropriate data (Activities 5 and 6);
- ◇ explain why, for earnings data, the mean is generally greater than the median (Activities 8 and 9);
- ◇ follow a procedure for investigating a problem or question;
- ◇ start to identify when and how you can use mathematical and statistical ideas to specify and investigate a problem.

3 How does occupation affect earnings?

Aims The main aim of this section is to investigate the link between occupation and men's and women's earnings, and to introduce some techniques for describing the *distribution* of a batch of data. ◇

3.1 Gender and occupation

Is the apparent difference between men's and women's earnings due to:

- ◇ women being paid less in the same occupation;
- ◇ women being employed predominantly in occupations which have relatively low pay;
- ◇ women not receiving 'equal pay for work of equal value';
- ◇ fewer women than men in senior, more highly-paid jobs?

The first two of these questions will drive the investigation in the next three sections. As you saw in the video in Section 1, establishing that one job is of equal value to another is difficult and problematic. The issue of whether women receive equal pay for work of equal value is beyond the scope of this investigation, as is the question of whether more men than women are in senior, highly-paid jobs.

So how could you set about investigating whether the difference between men's and women's earnings, observed in Section 2, is due to women being paid less than men for similar work, or to women being employed predominantly in lower-paid occupations? What sort of data are required?

Activity 10 *Desirable data*

If you were planning to investigate the relationship between occupation and men's and women's earnings, what sort of data would you like to have, and why? Think about this for a minute or two, then write down your ideas.

Activity 11 *Writing to learn mathematics*

Many of the activities in this and other units in the course ask you to respond by writing your ideas. What is the role of writing in learning mathematics? When and why will you need to write on this course?

You need to write to answer assignment questions. You may make notes as you work through the units. The Handbook activities require clear explanations of terms in your own words. Writing things down can also help you to form and clarify your ideas, and thus help you to learn.

In *Unit 1*, you were asked to describe your initial ideas about mathematics and the point was made there that this sort of exploratory writing is not easy. Look back to the notes or activity responses you have already completed. What would you say are some of the characteristics of such exploratory mathematical writing?

Writing is a skill that can be improved and learned. You are not expected to be a 'perfect' mathematical writer at this stage—but to become better at writing you need to practise. For many people, it takes quite some time to develop the skill of good writing about mathematical ideas. But you can begin here and consider a few points to help improve your skill.

When activities involve writing tasks, as well as responding to the activity as such, think about these questions:

- ◇ Why are you writing? (Apart from the obvious 'because the unit asked me to'!)
- ◇ Who do you feel you are writing for?
- ◇ How do you begin to write?
- ◇ What effect is your writing having on your thinking about the material?

As you consider these questions, see if you can also formulate a response to the question: 'What are some of the characteristics of good writing about mathematical ideas?'

What data are available?

The *New Earnings Survey* publishes the numbers of men and women that were included in the survey from each occupational group. The sample of employees included in the survey is assumed to be representative of all employees in the whole of the UK, so these numbers can be used to provide a good indication of the proportions of men and women employed in the various occupational groups listed in the survey. They will show whether certain jobs are done predominantly by men and others predominantly by women. Data are also available on both the weekly earnings and hourly earnings of men and women, either including or excluding overtime, for manual and non-manual workers as a whole. However, for individual occupations, data on earnings excluding overtime are available only for men (strange, but true!), whereas data including overtime are available for both women and men.

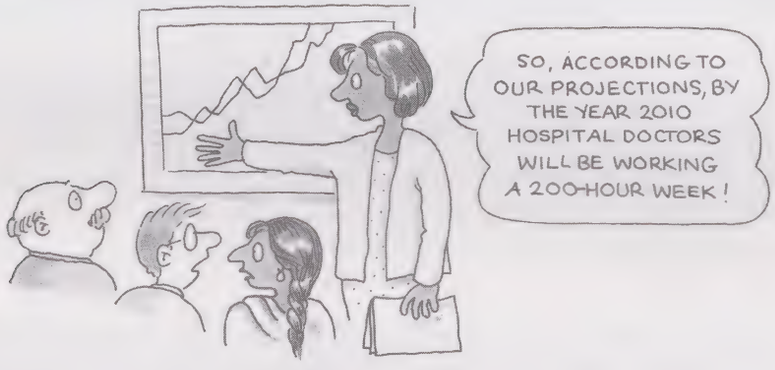
So looking at the individual occupational level will have to be a compromise on the ideal position (that is, comparing hourly earnings excluding overtime), and you will have to compare the earnings of men and women *including* overtime. (You might like to think about some likely effects of having to use these data.) However, it is possible to find out whether including overtime is likely to give rise to misleading results: data

are available on the average amount of overtime worked by men and women in individual occupational groups. If you find that, in a particular occupation, the average amount of overtime worked by men and women is very different, then you should treat the results with caution.

▶ Should you use data on weekly earnings or hourly earnings?

There are advantages to each. On the one hand, as has already been observed, men tend to work more hours per week than women, on average, and using hourly earnings eliminates the effect of this factor from our investigation. However, this difference in hours worked may be a feature of occupation rather than gender: you might expect men and women in the same occupation to work a similar number of hours each week. (You could check this as the data are available in the *New Earnings Survey*.) So using weekly earnings may be acceptable when comparing earnings *within* an occupation.

On the other hand, in many occupations, there is no paid overtime: pay is based on a nominal number of contracted hours and is fixed regardless of the number of hours actually worked, so that the published figure for hourly earnings may bear little relationship with reality. This is the case in many professions.



In secondary school teaching, for example, the basic working week according to the *New Earnings Survey* is approximately thirty hours. But according to a 1993 survey carried out by the Professional Association of Teachers, teachers in maintained secondary schools actually work about forty-two hours each week. And according to Professor Jim Campbell, who carried out a survey of teachers' workloads for the Association of Teachers and Lecturers, on average teachers work more than fifty hours a week: they 'put in more than forty hours on school premises and then spend a further eleven and a half hours on work, mainly at home'. In such occupations, official figures for hourly earnings are fairly meaningless and should certainly not be used in comparisons with earnings in other occupations.

Source: The *Guardian*, 12 July 1994.

Taking all these factors into account, and bearing in mind the data that are available, the course team settled for data on weekly earnings including overtime. But this will require the checking of overtime hours and total hours worked to see whether the weekly earnings figures given for men and women are truly comparable. We shall only use data for *hourly* earnings including overtime where the numbers of hours worked by men and women differ greatly within a given occupation.

As mentioned above, the *New Earnings Survey* includes information on the numbers of men and women in the various occupational groups included in the survey, and the proportions of men and women in each occupational group are assumed to be representative of the whole population of employees. So it is a relatively simple matter to find out whether there is evidence that women tend to work in lower-paid occupations.

First, look at the numbers of men and women in the survey who were working full-time in manual and non-manual occupations: the figures are given in Table 7.

Table 7 Numbers of men and women surveyed in manual and non-manual occupations

	Women	Men
Manual	7 305	32 771
Non-manual	34 136	40 108
Total	41 441	72 879

Source: *New Earnings Survey*, 1993, Tables D92 and D93

Activity 12 *Some basic facts contained in the table*

What percentage of the men surveyed were in manual occupations? What percentage were in non-manual occupations? Find the corresponding percentages for women. What do these figures tell you about the occupations of men and women? Also, to what extent does placing these figures in a table make it easier for you to process them?

The data indicate that a much smaller proportion of women than men work in occupations classified as manual, and a much larger proportion of women work in occupations classified as non-manual. The pie charts in Figure 7 represent the data given in Table 7. The areas of the circles are proportional to the numbers of men and women surveyed.

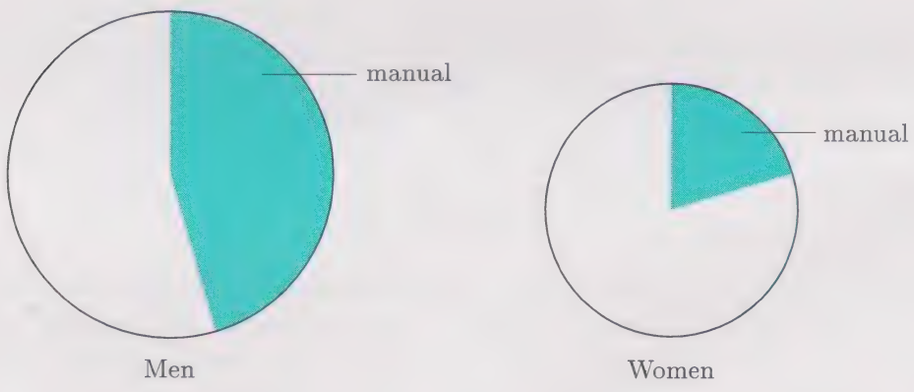


Figure 7 Pie charts showing the proportions of men and women surveyed in manual and non-manual occupations

Table 8 shows the numbers of women and men included in the survey in a variety of occupations. Even this small sample of occupations illustrates quite well that some jobs are done mainly by men and others mainly by women.

Table 8 Numbers of women and men in the survey in various occupations

	Women	Men
Chefs and cooks	301	417
Filing and records clerks	1068	525
Solicitors	132	173
Secondary teachers	1113	1044
Nurses	2293	385
Cleaners and domestics	579	542
Motor mechanics	0	1470

Source: *New Earnings Survey*, 1993, Tables D92 and D93

So it is possible that the difference between men's and women's earnings might, at least in part, be due to occupation. The next step is to compare earnings of men and women in the same occupation to see if there is evidence that women do earn less than men doing similar work.

Table 9 shows the median gross weekly earnings, including overtime, of adult men and women working full-time in six different occupational groups. The earnings ratio at the median is also shown for each group.

Table 9 Median gross weekly earnings including overtime (in pounds)

Occupation	Women	Men	Earnings ratio at the median (%)
Chefs and cooks	161	200	81
Filing and records clerks	193	245	79
Solicitors	399	541	74
Secondary teachers	389	443	88
Nurses	293	322	91
Cleaners and domestics	146	179	82

Source: *New Earnings Survey*, 1993, Tables D96 and D97

A check on the overtime worked and basic hours worked was carried out: the data are given in Table 10. (The figures given are averages; that is, means.)

Table 10 Normal basic hours and overtime for women and men

	Normal basic hours		Overtime	
Occupation	Women	Men	Women	Men
Chefs and cooks	37.3	39.8	1.3	1.6
Filing and records clerks	36.5	37.1	0.7	1.6
Solicitors	35.6	35.9	0.1	0.1
Secondary teachers	29.8	30.0	0.0	0.0
Nurses	37.3	37.6	0.4	0.9
Cleaners and domestics	37.0	39.1	1.5	4.0

Source: *New Earnings Survey*, 1993

Activity 13 Checking up on the hours worked

Comment briefly on the normal basic hours and overtime worked by men and women in the occupations given in Table 10. Do you think any of the differences are large enough to account for much of the difference in earnings between men and women in these occupations?

The data in Table 10 show that for secondary teachers and solicitors, there was very little difference in the official number of hours worked by men and women. Male nurses did half an hour more overtime per week on average than female nurses, but the normal basic hours worked were roughly the same. And male filing and records clerks had a slightly longer working week than the female clerks: their basic working week was just over half an hour longer and they did nearly an hour more overtime on average. However, these differences are small and are not large enough to account for the differences in earnings of men and women in these occupational groups.

The difference in working hours was greater in the other two occupational groups. The basic working week of chefs and cooks was two and a half hours longer for men than for women, although there was little difference in the overtime hours worked. For cleaners and domestics, both the normal basic hours worked and overtime were greater for men than for women (by over two hours each). These differences might well account for a large part of the difference in median weekly earnings of men and women in these two occupational groups. It is worth looking at the median hourly earnings for these two groups.

Table 11 shows the median gross hourly earnings (including overtime) for men and women in these two occupational groups.

Table 11 Median gross hourly earnings (in pence)

	Women	Men
Chefs and cooks	418	478
Cleaners and domestics	394	427

Activity 14 The median hourly earnings

Calculate the earnings ratio at the median, based on gross hourly earnings, for men and women in both occupational groups in Table 11. State the effect that using hourly earnings data instead of weekly earnings data has had on the earnings ratio at the median.

Using hourly earnings data instead of weekly earnings data increased the earnings ratio at the median for both occupational groups. What effect

would excluding overtime have in each case? First, look at the effect that working overtime has on overall hourly earnings.

If overtime is excluded, then the earnings per hour of an individual is just the normal hourly rate. However, if overtime is included and overtime rates are greater than normal hourly rates, then the more overtime someone works, the higher their overall earnings per hour will be. The next activity should help you to see how this occurs.

Activity 15 *The effect of including overtime*

Ivor is paid £4 an hour for a fixed forty-hour week. When he works overtime he is paid £6 an hour for each extra hour. Draw up a table to show his total earnings and overall hourly rate when he works different numbers of hours between forty and fifty hours in a week. (Decide whether you are going to work out the figures for every extra hour of overtime, or some reduced set of hours.)

The results of the last activity illustrate how overall hourly earnings rise as the amount of overtime worked goes up. Since male and female chefs and cooks work similar amounts of overtime, excluding overtime would reduce their hourly earnings by similar amounts, so this would have very little effect on the earnings ratio at the median for this group. On the other hand, male cleaners and domestics do far more overtime on average than female cleaners and domestics. So, provided overtime rates are greater than normal hourly rates, excluding overtime would reduce the hourly earnings more for the men than for the women. Therefore the earnings ratio at the median would increase. So greater overtime could account for some or all of the difference between the hourly earnings of male and female cleaners and domestics.

In summary, the earnings ratios at the median in Table 9 vary between 74% for solicitors and 91% for nurses. A brief investigation into hours worked has suggested that the figures might be misleading for two of the occupational groups: chefs and cooks, cleaners and domestics. In Activity 14, you calculated the earnings ratios at the median for these two groups based on hourly earnings data: these were 87% for chefs and cooks and 92% for cleaners and domestics. And it was observed that if overtime rates are greater than normal hourly rates, then overtime could account for some or all of the difference between men's and women's hourly earnings for cleaners and domestics. Nevertheless, it seems that women are paid less than men 'on average' in all these occupations.

The Equal Pay Act, which was passed in 1970, was fully implemented in 1975. It provided for equal pay for men and women doing work of 'a broadly similar nature'. Yet, for the occupations looked at in our investigation, it appears that women are paid less than men on average.

Activity 16 *Finding and communicating an explanation*

Assuming that employers are not breaking the law and paying different wages to their male and female employees, can you suggest a possible explanation for this? Write a sentence or two giving your views and explanation.

Before you move on to the next subsection, take time to think back to the earlier questions posed about writing. The last few activities have involved you in responding by writing. So, in these activities, what is your purpose in writing? Who are you writing for? How did you begin to write?

The activities were intended to help you in your learning of the ideas introduced, so you could have said the writing was to help you keep your learning 'active' (not least, to stop you falling asleep!), and to help you think about and put down ideas in your own words. You were perhaps writing in this case for yourself alone and not for any other audience. How did you begin to write? You may have sat and thought about what you might say; you might have jotted some ideas down and then started to write the response; you may have talked it over with someone else or asked them to look it over and comment afterwards. Look back at your responses. How have they helped you in your learning of the ideas presented in this section? Do you feel that writing something down helps you learn? Think about this last point as you complete future activities.

3.2 The distribution of earnings

For each of the six occupations considered in Subsection 3.1, the data indicate that women are paid less than men on average. But does that mean, for example, that all female solicitors are paid less than all male solicitors, and all female nurses are paid less than all male nurses? This seems very unlikely. To discover more about the relative earnings of men and women you need information on the *distribution* of earnings, not just on average earnings: solicitors do not all earn exactly the same as each other nor do nurses. Using average earnings simplifies comparisons, but to gain a greater understanding of how women's earnings compare with men's earnings, you need to look at the distribution of earnings.

Some data are available in the *New Earnings Survey* on the distribution of earnings within occupations. However, these data are all in the form of *summary statistics*; that is, numbers which summarize the data, such as the mean and the median. Several other summary statistics are included, as you will see in Section 5. However, first you need to know what these statistics are. And to understand them, you need to know how they are calculated.

There are several different ways of measuring the spread of the earnings within a group. To illustrate them requires data on the earnings of individuals, but unfortunately these are not readily available. However, the *New Earnings Survey* contains a good deal of information about the general distributions of earnings within occupations. This has been used to construct some sets of data to use in this subsection and in the Appendix. Although these batches do not contain real data, the values in them will be taken as being representative of the earnings of individuals.

The data in Table 12 on the earnings of twenty-two solicitors (eleven male and eleven female) were made up from information on the distribution of earnings contained in the 1993 *New Earnings Survey.* These data will be used to introduce you to two of the different measures of spread (two more are described in *Unit 4*).

Table 12 The gross weekly earnings of twenty-two solicitors (in pounds)

Women	238	273	324	348	387	405	430	555	580	672	808
Men	283	346	370	452	522	543	595	650	762	907	1080

The range

The *range* of a batch of data is the numerical difference between the smallest or minimum value, called *min*, and the largest or maximum value, called *max*.

$$\text{range} = max - min$$

Activity 17 Calculating the range

Find the range of the batch of women's earnings in Table 12. Find the range of the batch of the eleven men's earnings. Is the range greater for the women or for the men?

Quartiles and the interquartile range

Given a batch of data in numerical order, such as either of the two batches in Table 12, the range can easily be calculated, but it reveals nothing about how the values in the main body of the data are distributed. Most of the values could be bunched around the median, or they could be spread fairly evenly between the minimum and maximum values, *min* and *max*, or they might be irregularly distributed anywhere between *min* and *max*.

The range is also very sensitive to changes in the two extreme values, *min* and *max*. Suppose, for example, that the highest-paid of the female solicitors was an exceptionally talented and successful woman earning, say, £2000 a week, instead of £808. Then the range of the batch would have been £(2000 − 238) = £1762, giving the impression that the earnings of the women solicitors were very widely spread. In fact, apart from this one

value, all the values in the batch were within £434 of each other. A measure of spread which is not influenced by extreme values in this way would be useful.

What is needed is a measure which conveys information about the spread of values *in the main body of the data.*

Activity 18 Assessing some measures of spread

Here are three possible measures of spread. What are some strengths and weaknesses of each measure? Can you suggest an alternative measure?

(a) Exclude the two extremes and find the range of the remaining values.

(b) Exclude five values from each end of the batch and find the range of the remaining values.

(c) Include only the first five values each side of the median and find the range of these values.

What is required is a measure that takes into account the size of the batch of data. One such measure is based on the difference between two particular values in the batch, known as the *quartiles.* As the name implies, the two quartiles lie approximately one quarter of the way into the batch from either end. Roughly speaking, 25% of the values in a batch lie below the *lower quartile* and 25% of the values lie above the *upper quartile*: roughly speaking, the quartiles enclose between them the middle 50% of values. This is illustrated in Figure 8.

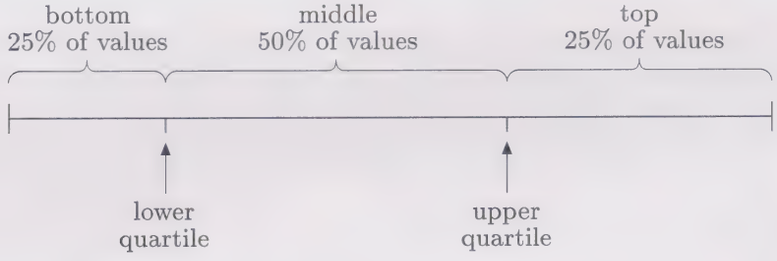


Figure 8 The upper and lower quartiles of a batch of values

Notice the use of the word 'approximately' and the phrase 'roughly speaking'; to see why they are there, look at this example. Suppose, for instance, that there are 10 numbers in a batch. Now 25% of 10 is 2.5, which is not a whole number. So it is not quite accurate to say that exactly 25% of the values lie below the lower quartile. You will see shortly how the quartiles are found in cases like this.

There are a number of subtly different criteria for deciding which values in a batch to use to find the quartiles: the criterion described in this course is not the only one. You may well come across a slightly different one at some time—in another course or in a book or when using a statistics package on a computer—so you should be aware that there are other possibilities.

However, whichever method is used, the results obtained are similar, although small differences in the actual values of the quartiles may occur.

In the calculator work that follows, you are asked to investigate the rules that your calculator uses to find the quartiles of a batch of data.

Now work through Section 3.1 of Chapter 3 of the Calculator Book.

Finding the quartiles of a batch is very similar to finding the median. First, sort the batch into order, smallest first. Then the median is the middle value, or the midpoint of the two middle values if there is an even number of values in the batch.

The *lower quartile*, which is denoted *Q1*, is the median of the lower half of the batch of data—that is, of the values to the left of the median in your list. The *upper quartile*, which is denoted *Q3*, is the median of the upper half of the batch of data—that is, of the values to the right of the median.

With this new labelling, it makes sense, retrospectively, to think of the median as *Q2*, though this is not standard notation.

Here is a notational device that may help you picture these ideas. Imagine the letter 'W' with the median and quartiles at the hinges. For example, for a batch of size seven, the configuration would look like Figure 9.

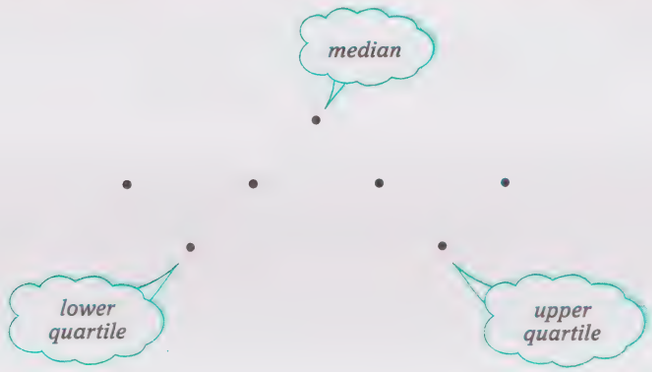


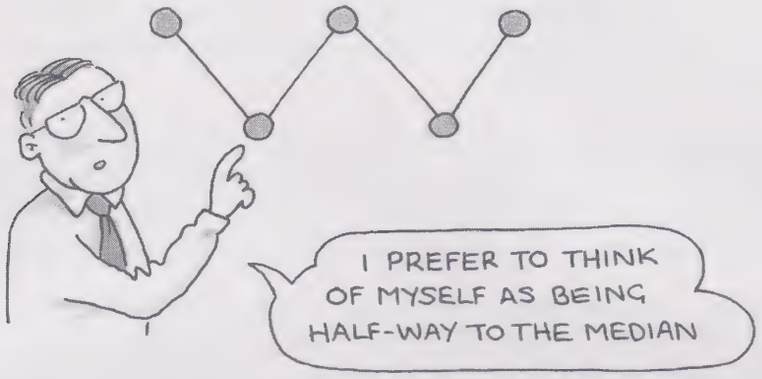
Figure 9 W-shaped diagram for a seven-item batch

In this case, there is an odd number of values in the batch, so the median is the middle value. There are three values in the lower part of the batch, to the left of the median. The lower quartile is the middle one of these three values. The upper quartile is the middle one of the three values to the right of the median.

For a batch of size eight, the diagram looks like Figure 10.

Figure 10 W-shaped diagram for an eight-item batch

In this case, there is an even number of values, so the median is halfway between the two middle values. Hence there are four values in the lower half (to the left of the median). The lower quartile is the median of these four values, and is therefore halfway between the second and third values. The upper quartile is found in a similar way by finding the median of the four values in the upper half of the batch.



Activity 19 Drawing diagrams

(a) Draw a W-shaped diagram to represent a batch of nine values and mark on it the positions of the median and quartiles.

(b) Draw a W-shaped diagram to represent a batch of ten values.

Activity 20 Using diagrams

(a) Draw a W-shaped diagram to represent a batch of eleven values.

(b) Use your diagram to find the median and quartiles of the batch of men's earnings in Table 12.

(c) Use your diagram to find the median and quartiles of the batch of women's earnings in Table 12.

Activity 21 Thinking about diagrams

We have suggested a diagram—the W-shaped diagram—that may clarify an idea. Stop and think for a moment. Did it help? If so, how and why? If not, what got in the way or confused you? Recall the phrase 'stressing and ignoring' from *Unit 1*. Any diagram will stress certain things at the expense of ignoring others. What has been highlighted by this diagram and what has been left out or downplayed? Is calling it the 'W-shaped diagram' helpful, or would a name related to what it is to be used for be better?

There is a measure of spread based on the quartiles.

> The *interquartile range* is the numerical difference between the upper and lower quartiles: $Q3 - Q1$.

Example 3 Interquartile range

Find the interquartile range for the batch of men's earnings in Table 12.

$Q3 - Q1 = £762 - £370 = £392.$

Activity 22 Finding the interquartile range

Find the interquartile range for the batch of women's earnings in Table 12. Is this greater or smaller than the interquartile range for the men's batch?

Activity 23 Comparing the spreads

(a) For the data given below in Table 13, find the range and interquartile range of the men's earnings and of the women's earnings. (Use your calculator to sort the data into ascending order.)

(b) Briefly compare the spread of the earnings of male and female chefs and cooks.

Table 13 The gross weekly earnings of seventeen chefs and cooks (in pounds)

Women	165	210	110	235	152	128	172	136	
Men	147	275	233	188	165	330	130	200	249

Two measures of spread (the range and the interquartile range) have been introduced in this subsection; these have been used to compare the spread of batches of earnings of men and women in two occupations (solicitors; chefs and cooks). In both cases, the spread was greater for the men than for the women. Although, for these batches, some of the women earned more than some of the men, the earnings of the men were generally higher than the earnings of the women.

Before you finish this section, look back at your study schedule for this unit and the targets you set. Do you wish to make any adjustments?

Outcomes

After studying this section, you should be able to:

- ◇ extract relevant information from tables of data and written descriptions (Activities 12, 13, and 14);
- ◇ describe some of the advantages and disadvantages of different measures of spread (Activity 18);
- ◇ explain the meaning of the terms 'range', 'lower quartile', 'upper quartile' and 'interquartile range' and find the values of each for a given batch of data (Activities 17, 20, 22 and 23);
- ◇ find the median and the quartiles of a batch, by using of a W-shaped diagram, or otherwise, and comment on some of the features of this particular diagram (Activities 19, 20 and 21);
- ◇ produce short written descriptions and explanations exploring certain economic and social issues (Activity 16).

4 Boxplots

Aims The main aim of this section is to introduce the boxplot as a way of representing the distribution of a batch of data and to show how boxplots can be used to compare batches of data. ◇

In the previous section, you saw that, in each of the six occupations looked at, the median earnings figure of men was higher than the median earnings of women. This prompted the question: 'Is there a difference between men's and women's earnings across the whole range from the lowest-paid to the highest-paid?'

The median and the quartiles were used to provide information about the level of earnings in a batch, and the range and interquartile range were used to measure the spread of earnings. In this section, a diagram which contains all this information is introduced: the boxplot. The boxplot's major strength is that it allows you to make a direct visual comparison of two batches of data at once. As you probably noticed in *Unit 2*, it is often easier to interpret information which is presented in a diagram than to extract and interpret information from a list of numbers.

4.1 What is a boxplot?

The W-shaped diagram shows the positions of the key values of a batch of data. It is made up from five important points which help to summarize the distribution (or shape) of a batch of data: the median (which you might like to think of as $Q2$), the two quartiles ($Q1$ and $Q3$) and the two extreme values (min and max).

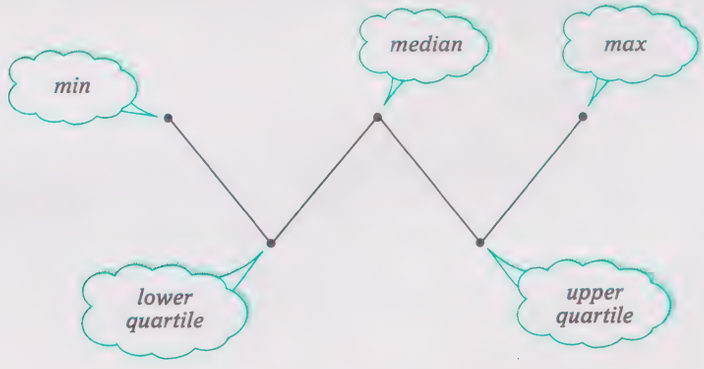


Figure 11 The W-shaped diagram in general

The five values at the five key positions of the 'W' can be presented in a different diagram, called a *boxplot*. For the eleven earnings values for male solicitors given in Table 12, the boxplot looks like Figure 12.

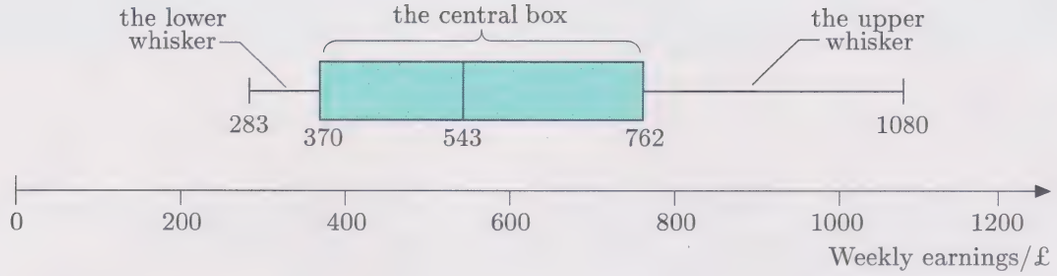


Figure 12 Boxplot of earnings of eleven male solicitors

The central feature of this diagram is a *box* which extends from the *lower quartile* to the *upper quartile*. This part of the diagram contains approximately 50% of the values in the batch. The length of the box is equal to the *interquartile range*.

Boxplots are sometimes called *box and whisker diagrams*.

Outside the *box* are two *whiskers* extending from each quartile to the corresponding extreme value. Each whisker covers 25% of the remaining batch. The distance between the extreme ends of the two whiskers is equal to the *range*.

So far you have seen how the two quartiles and the two extremes are represented on the boxplot. The median is shown by putting a vertical line through the box. Since the median may be nearer one quartile than the other (as in Figure 12), the vertical line representing it will not necessarily be centrally placed in the box.

Thus, a boxplot shows clearly the division of a batch of data into four parts: the two whiskers and the two sections of the box. These sections correspond to the same four parts of the W-shaped diagram and each contains (approximately) 25% of the values in the batch.

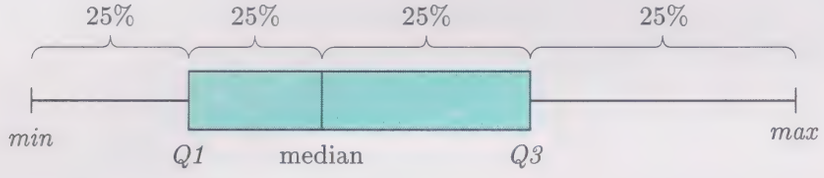


Figure 13 A typical boxplot

The boxplot in Figure 13 is a fairly typical one in shape: most batches of data are more densely packed in the middle of the batch and less dense near the extreme values. This means that each whisker is usually longer than half the length of the box (or equivalently, the two whiskers together are usually longer than the box). This is illustrated in the next example.

Example 4 Boxplot for female solicitors' earnings

Figure 14 shows the boxplot for the gross weekly earnings of the eleven female solicitors given in Table 12 (page 35).

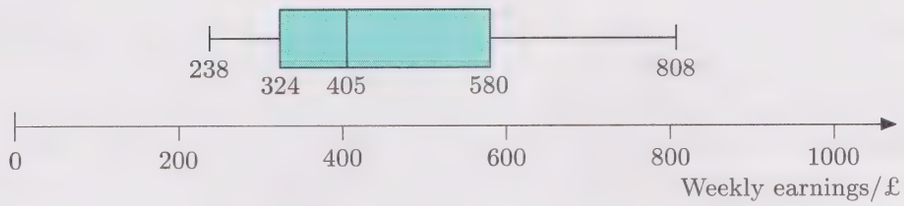


Figure 14 Boxplot of earnings of eleven female solicitors

As you can see, the whiskers are longer than the corresponding sections of the box, indicating that the values are more widely spread out in the whiskers than in the box.

Notice that, in both Figure 12 and Figure 14, the five values—the median, the two quartiles and the two extremes—are written on the boxplot. This is customary, so remember to include them whenever you draw a boxplot.

4.2 Comparing batches using boxplots

A boxplot can be a very effective way of presenting information about the distribution of a batch of data in a diagram. However, it really comes into its own when used for comparing two or more batches of data. If boxplots for two batches are drawn on the same diagram, with the same scale, then a direct visual comparison of the two batches can be made.

Figure 15 shows, on the same diagram, the boxplots for the male and female solicitors of Table 12.

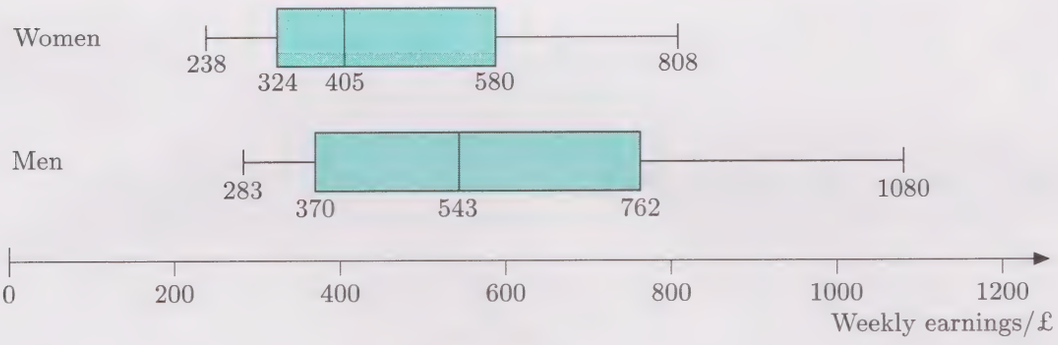


Figure 15 Gross weekly earnings of solicitors

Activity 24 Comparing earnings using boxplots

Use the boxplots in Figure 15 to compare the earnings of the male and female solicitors.

Before discussing how to use your calculator to obtain boxplots, there is one other point worth noting about the batches of earnings data shown in Figure 15. The batches are not symmetric: for each boxplot, the right-hand whisker is much longer than the left-hand whisker, and the right-hand section of the box is longer than the left-hand section: the boxplots are *asymmetric*. This reflects the fact that the higher earnings values are more widely spread than the lower values. This is frequently the case with earnings data: remember the image of the pay parade (Activity 9 in Section 2).

A batch of data which exhibits this type of asymmetry—one whisker of the boxplot and the corresponding section of the box being longer than the other—is said to be *skewed*. Earnings data are generally *right-skewed*, since a boxplot of earnings data usually exhibits a long *right* whisker and *right*-hand section of the box. Whenever the boxplot for a batch of data has a long right whisker, the mean is usually larger than the median. In particular, the mean of a batch of earnings data will generally be higher than the median. (Recall the discussion about the mean and the median of earnings data in Section 2.)

WHEN ONE WHISKER IS LONGER THAN THE OTHER, THE DISTRIBUTION IS SKEWED

Of course, not all batches of data, not even all batches of earnings data, are right-skewed. A batch may be symmetric—in this case, the two whiskers will be roughly the same length and the two sections of the box will be roughly the same length (as in Figure 16(a)). Or a batch may be *left-skewed*—this is the case when the left whisker is longer than the right whisker and the *left-hand* section of the box is longer than the right-hand section (as in Figure 16(b)).

(a) Symmetric

(b) Left-skewed

(c) Box suggests left-skewed but tails suggest right-skewed

Figure 16 Three boxplots

Of course, the boxplot for a batch of data may not fit any of these descriptions; for example, it may have a longer right whisker than left, but a longer left-hand section of the box (as in Figure 16(c)). In such a situation, you need to make it clear whether you are talking about the box or the whiskers of the boxplot when describing the shape of the data.

Now work through Section 3.2 of Chapter 3 of the Calculator Book.

Activity 25 *Comparing earnings for chefs and cooks*

A sketch of the boxplots for the chefs' and cooks' earnings data from Table 13 (page 39) is shown in Figure 17.

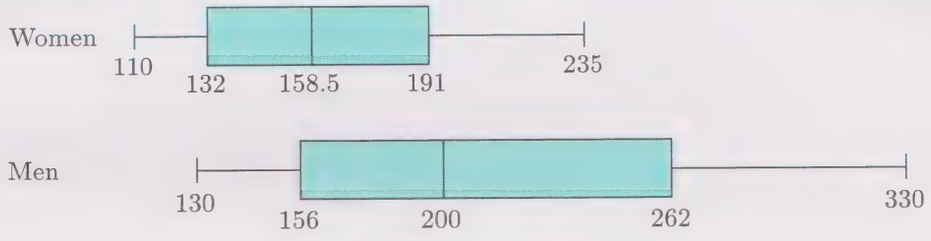


Figure 17 Sketch of boxplots of earnings data for chefs and cooks

(a) What do the boxplots tell you about the relative earnings of male and female chefs and cooks?

(b) What do the boxplots tell you about the shapes of the distributions of the earnings of male and female chefs and cooks?

(c) Make some observations about how boxplots helped you to interpret the data presented.

Detailed information and advice on the drawing of boxplots, as well as some further activities using the data from this section, is in the Appendix 'Drawing accurate boxplots' on page 70. If you are happy about how to draw a boxplot by hand, just do Activities 44–46 from this appendix now. Otherwise, you may wish to study the material in the Appendix now.

When the boxplots for two batches of earnings data are drawn on the same diagram to the same scale, a visual comparison of the level and distribution of the earnings in the two batches can be made. You have probably found it easier to compare two boxplots than to compare two lists of numbers. In this section (and in the Appendix), boxplots have been used to compare the distribution of the earnings of men and women in four occupational groups. In all four cases, they showed that the level of the men's earnings was generally higher, although the difference was much smaller for the nurses (see the Appendix for details) than for the other three groups.

Outcomes

After studying this section, you should be able to:

- ◇ compare batches of data using boxplots (Activities 24 and 25);
- ◇ use your calculator to display a boxplot;
- ◇ identify the median, the lower quartile, the upper quartile, the smallest and largest values of a batch of data, given a boxplot (Activities 24 and 25);
- ◇ draw a rough sketch of a boxplot (Activities 45 and 46);
- ◇ draw a boxplot accurately by hand (Activities 44, 45 and 46);
- ◇ interpret a boxplot (Activities 25, 45, and 46).

5 Interpreting and representing earnings data

Aims The main aim of this section is to show how boxplots are used to represent summary statistics from the *New Earnings Survey*, and to show how boxplots and earnings ratios may be used to compare the earnings of men and women in a number of occupations. ◇

This section ends the investigation into the earnings of men and women, and in particular into the relationship between gender and occupation. The batches of earnings data used in Section 4 were constructed from information contained in the *New Earnings Survey*. Some of the summary statistics which are available in the *New Earnings Survey* (which were used to construct the data) will be described in this section.

Data on the distribution of earnings in a large number of occupational groups is published in several forms in Volume D of the *New Earnings Survey*. One set of tables gives, for each occupation, the percentages of men and women in that occupation earning less than £170, £190, £220, £250, £270, £300, £340, £400, £470, £550 and £700. Another set of tables gives information such as the median and the quartiles for men and women in each occupation.

Table 14 Distribution of gross weekly earnings of solicitors in 1993 (in pounds)

	Women	Men
Upper quartile	588	760
Median	399	541
Lower quartile	329	365

Source: *New Earnings Survey*, 1993, Tables D96 and D97

Activity 26 *Interpreting the table*

(a) What percentage of the men referred to in Table 14 earned between £365 and £760?

(b) What was the largest amount earned by any of the lowest-paid 25% of the women?

(c) Approximately what percentage of the men earned more than £760 during the week of the survey? Approximately what percentage of the women earned less than £588 per week?

Deciles

The lowest and highest *deciles* of a batch of data provide information about the tails of its distribution. Just as the quartiles cut off a quarter, or 25%, of the values at either end, the *highest decile* cuts off the top tenth or 10% of values, while the *lowest decile* cuts off the bottom 10%. This is illustrated in Figure 18.

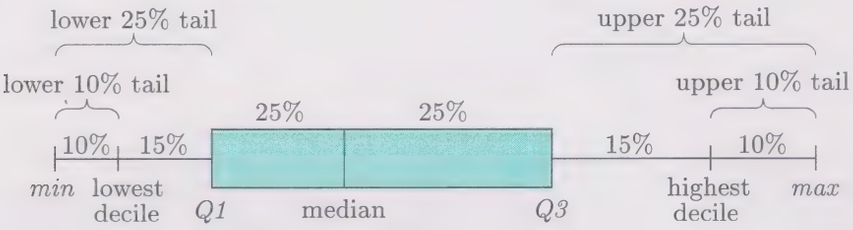


Figure 18 The highest and lowest deciles on a boxplot

Table 15 includes these summary statistics for male and female solicitors in addition to the information given in Table 14.

Table 15 Distribution of gross weekly earnings of solicitors in 1993 (in pounds)

	Women	Men
Highest decile	750	1063
Upper quartile	588	760
Median	399	541
Lower quartile	329	365
Lowest decile	253	301

Example 5 Interpreting summary statistics

Interpret the figures in Table 15 for male solicitors.

The median earnings of male solicitors was £541. This means that, in the week of the survey, 50% of this group of male solicitors earned more than £541 while 50% earned less than £541. The upper and lower quartiles in this column are £760 and £365, so 25% of this group earned less than £365 and 25% earned more than £760. The extra information obtained by looking at the highest decile (£1063), and the lowest decile (£301), is that 10% of this group earned more than £1063 and 10% earned less than £301. Note that you are not told the values of the highest and lowest amounts earned by any individual in this group.

Of course, if one of the men earned exactly £541, the median earnings, then the percentage earning more than £541 would not be exactly 50%, but only approximately 50%. Similar comments apply if someone's earnings are exactly equal to the lowest decile, or the lower quartile, etc.

Activity 27 Interpreting Table 15

(a) What percentage of the female solicitors included in the survey earned £750 or more?

(b) What percentage of the female solicitors earned £253 or less?

(c) What percentage of the female solicitors earned between £399 and £750?

(d) What percentage earned between £253 and £329?

Decile boxplots

A boxplot can provide a helpful diagrammatic representation of a batch of data. It is usually very much easier to grasp the distribution of a batch of data or to compare two batches of data by looking at boxplots than by studying lists of numbers. It would therefore be informative to draw a boxplot for the type of data in Table 15. However, the boxplot cannot be exactly like those drawn in Section 4, since there the extremes of a batch, *min* and *max*, were marked on the boxplot, but you do not know the extremes for the batches described by the statistics in Table 15.

Recall that if the extreme values in a batch are unusually large or small, including them may give a distorted picture of the main body of the data: this was the reason for preferring the interquartile range to the range as a measure of spread. When dealing with large batches of data from surveys like the *New Earnings Survey*, it is common not to be given the extreme values, but to be given the highest and lowest deciles instead. The deciles are not as affected by unusual values in the way that the extremes (*max* and *min*) are. So a boxplot is often drawn which extends only from the lowest decile to the highest decile. A boxplot like this is called a *decile boxplot*.

Example 6 Decile boxplots for solicitors' earnings

Figure 19 shows decile boxplots for the weekly earnings of the male and female solicitors included in Table 15.

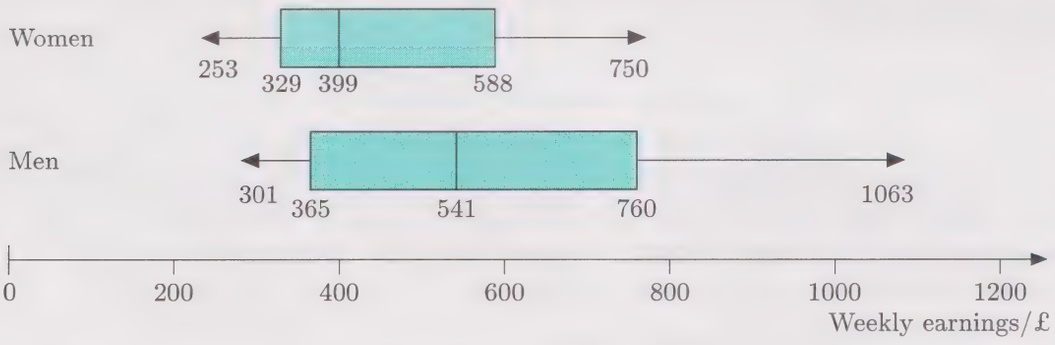


Figure 19 Decile boxplots of solicitors' earnings

Notice that arrowheads are used at the ends of the whiskers instead of vertical bars to indicate that these points represent the highest and lowest deciles and *not* the extremes. The arrowheads point to the missing 10% of the values on either side.

Activity 28 Interpreting decile boxplots

What do the boxplots in Figure 19 tell you about the relative earnings of male and female solicitors?

The boxplots in Figure 19 provide a useful visual summary of the information contained in Table 15. It is immediately clear that the spread of the earnings is greater for the men than for the women, both within the box and in the whiskers. Earnings are higher for men than for women across the whole range.

Now consider male and female chefs and cooks. The basic working week is longer for male chefs and cooks than for female chefs and cooks. So, in order to compare like with like, use data on *hourly* earnings. Table 16 contains earnings data from the 1993 *New Earnings Survey.*

Table 16 Distribution of gross hourly earnings of chefs and cooks in 1993 (in pence)

	Women	Men
Highest decile	594	763
Upper quartile	501	620
Median	418	478
Lower quartile	375	385
Lowest decile	333	325

Activity 29 Drawing and interpreting decile boxplots

Draw decile boxplots for these data and use them to compare the levels and distributions of the earnings of male and female chefs and cooks. Write a paragraph summarizing what you have found out.

Look back to the earlier paragraphs on characteristics of writing and look at your own responses. Could another student understand your writing?

Earnings ratios

In Section 2, earning ratios at the mean and at the median were used to compare the average earnings (mean and median) of men and women. They can also be used to compare earnings over the whole range from the lowest-paid to the highest-paid. The earnings ratios at the quartiles and at the highest and lowest deciles are defined in a similar way to the earnings ratios at the mean and median. For example, the *earnings ratio at the lower quartile* is calculated as follows.

$$\frac{\text{lower quartile earnings of women}}{\text{lower quartile earnings of men}}$$

The *earnings ratio at the highest decile* is calculated as follows.

$$\frac{\text{highest decile earnings of women}}{\text{highest decile earnings of men}}$$

The earnings ratios at the median, quartiles and deciles for the solicitors of Table 15 are given in Table 17.

Table 17 Earnings ratios for solicitors

Highest decile	71%
Upper quartile	77%
Median	74%
Lower quartile	90%
Lowest decile	84%

Recall from page 16 that earnings ratios are often expressed as percentages: to convert the figure from the above formula, merely multiply by one hundred.

These figures confirm that the earnings of female solicitors are lower than the earnings of male solicitors over the whole range, particularly at the higher end: the earnings ratio is only 74% at the median, 77% at the upper quartile and 71% at the highest decile. Remember, a low earnings ratio (considerably lower than 100%) indicates a large 'gap' (measured relatively) between women's and men's pay. The lower-paid women's earnings are closer to those for the men: lower quartile women earn 90% of the amount earned by lower quartile men.

Activity 30 Earnings ratios

Calculate the earnings ratios at the median, the upper and lower quartiles and the highest and lowest deciles for the chefs and cooks in Table 16. Comment on what these figures tell you about the relative earnings of male and female chefs and cooks.

In this section, decile boxplots and various earnings ratios have been used to compare the distributions of the earnings of men and women in various occupations. Decile boxplots provided an informative way of representing visually some of the data from the *New Earnings Survey*. Earnings ratios allowed the measurement of the relative earnings of women and men at different points of the two distributions.

▶ What has been discovered about the relative earnings of men and women?

In Section 2, it was found that in 1993 women received, on average (excluding overtime), only about 80% of the amount paid to men for an hour's work.

The last three sections have been concerned with investigating whether this difference between the earnings of men and women is due to women being paid less for similar work, or to women being employed predominantly in occupations which have relatively low pay. Data on the numbers of men and women in several occupations confirmed that the proportions of male and female workers vary from occupation to occupation, so this could account for some of the difference in earnings. However, in each of the occupations for which data were obtained, the men's earnings were generally higher than the women's.

How is this to be interpreted? Does this mean that men and women do not receive equal pay for similar work? Or are there still other factors that have not been taken into account? For example, might the difference in earnings within an occupation be due to men obtaining earlier promotion than women—or to more men than women obtaining promotion overall?

These are much more difficult questions to answer and the data needed to answer them are unfortunately not readily available.

▶ So what can be concluded?

It seems that men do earn more than women on average, and at least some but not all of the difference may be accounted for by factors such as occupation and hours worked. However, a much more detailed investigation would be required to determine the reasons for the difference between the earnings of men and women *within* an occupation. As yet, no firm conclusion can be reached from this investigation about whether men and women receive equal pay for equal work.

Outcomes

After studying this section, you should be able to:

- ◇ explain the terms 'highest decile' and 'lowest decile' (Activity 27);
- ◇ draw decile boxplots to represent earnings data given in the form of summary statistics, and make comparisons between distributions using decile boxplots (Activities 28 and 29);
- ◇ calculate and interpret quartile and decile earnings ratios (Activity 30).

6 The growth of low pay

Aims The main aim of this section is to interpret critically the information contained in a reader article on earnings. ◇

This section consists of a series of activities based on the article 'No let-up in the growth of low pay'. The article was first published in *The New Review* No. 25 in January 1994.

Read the article through to get a sense of the points that it makes and the arguments it uses to make these points. Read the article critically. *The New Review* is a publication of the Low Pay Unit. You may or may not agree with the spirit in which the article is written, but in either case watch out for unsubstantiated claims or inaccuracies.

The Low Pay Unit, which is both a charity and a limited company, runs an employment rights service aimed at helping those who are low-paid or unfairly treated at work. The unit is also a campaigning organization for a minimum wage and the Social Chapter as enshrined in the Maastricht agreement of the European Union.

After studying the earlier sections of this unit, much of the information contained in this article will be described in terms now familiar to you. Although quite short, the article contains a lot of information, so do not expect to take in all the details in a single reading. Read the article through once now, then reread the relevant sections as you work through the activities.

There are some details in the article that are not fully explained. For example, the article refers to the Council of Europe's decency threshold: this is defined to be 68% of mean gross weekly earnings (based on a thirty-seven and a half hour week). This means that anyone earning less than 68% (roughly the 'two-thirds' mark) of mean gross weekly earnings is considered to be earning less than a 'decent' wage.

Reread the article 'No let-up in the growth of low pay'.

Activity 31 *The decency threshold*

The mean gross weekly earnings including overtime was £316.90 in 1993 and the mean gross weekly earnings excluding overtime was £299.80. Which of these two figures has been used to calculate the decency threshold figure of £215.50 given in Table 1 in the article?

Activity 32 *Checking the accuracy of the figures*

In 1993, the average number of hours worked per week (for all full-time employees) was approximately thirty-seven and a half excluding overtime and forty including overtime. Given this additional information, decide whether or not you are convinced about the accuracy of the figures quoted for Europe's decency threshold (£215.50 per week, £5.75 per hour in 1993/4) in Table 1 in the article? If you are not, then briefly explain why.

Leaving aside any reservations you might have about the numbers in Table 1, try the following activities.

Activity 33 Interpreting Table 1

(a) How has the number of people with gross weekly earnings below the decency threshold changed between 1979 and 1993?

(b) Have men and women fared equally well over this period?

(c) The percentages in Table 1 refer to the percentage of all full-time workers in each category. For example, in 1993, 50.5% of all women in full-time employment earned less than the 1993 decency threshold. In 1993, there were roughly equal numbers of men and women with earnings below the decency threshold. Why might the percentage figure for women be so much higher than the percentage figure for men?

Activity 34 Interpreting Table 3

(a) Use the information in Table 3 to estimate the lowest decile and the highest decile of the distribution of gross weekly earnings for full-time male manual workers in 1993.

(b) What is the significance of the fact that the numbers in the final column are increasing?

Activity 35 Interpreting Tables 6 and 7

(a) What effect does excluding overtime and using hourly earnings instead of weekly earnings have on the earnings ratio? How do you account for the difference in the results obtained from these two methods of calculation?

(b) It is not possible to tell from the information given in the article whether the mean or the median has been used in Tables 6 and 7. Do you think this matters?

Activity 36 Looking for evidence

(a) According to the article: 'The gap between men's and women's pay remains largely untouched.' What evidence is given in the article to support this claim? Is there any evidence to the contrary?

(b) Later in the same paragraph, the following statement is made: 'This marks the end of progress towards equality which has been in train over the last few years.' Is there any evidence for this in the article?

Activity 37 *Tracking an error*

(a) In the discussion of young people's earnings at the end of the article, the author refers to 'a drop of 11.9 percentage points'. What do you think is meant by 'a drop of 11.9 percentage points'?

(b) What information would you need in order to check the statement that a drop of 11.9 percentage points represents £34.54 a week in today's money?

(c) The figure you need to carry out the check mentioned in part (b) is given in one of the tables in the article. Find this figure.

(d) Find 11.9% of the figure in part (c). Check carefully the other figures in the paragraph containing the statement. Can you find an error?

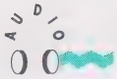
You have now looked closely at quite a lot of information contained in the article. If you found it difficult to follow some of the details on your first reading, then try reading it through again and assess what you have gained by working through the activities. You may find it helpful to note what you found particularly difficult or hard to follow. Did such details contain claims involving data from the tables, or perhaps unsubstantiated claims? If you came across a similar article in the future, how confident do you feel that you would be able to understand a good deal of it? And would your attitude to accepting at face value any claims contained in such an article have changed as a result of working on this article?

Outcomes

After studying this section, you should be able to:

- ◇ extract and interpret information from an article on earnings (Activities 31, 33, 34 and 35);
- ◇ recognize certain unsubstantiated claims and inaccuracies (Activities 32, 36 and 37);
- ◇ start to evaluate written economic articles in newspapers in terms of their statistical strengths and weaknesses, and comment on the validity of their calculations and claims (Activities 31 to 37).

7 Changes in earnings



Aims The main aim of this section is to discuss what the Average Earnings Index measures and how it may be used in conjunction with the Retail Prices Index to investigate the question 'Are people getting better off?' ◇

Part of a trade union's job is to try to protect and improve the standard of living of its members. Some years ago, when asked about his union's forthcoming pay negotiations, Gavin Laird (General Secretary of the Amalgamated Engineering Union) replied: 'We will be insisting on increases in line with inflation at the very least.' How often have you heard similar statements? You may have noticed that pay claims tend to rise and fall with inflation. People generally feel that if they are to be 'better off', then they must obtain pay rises above the increase in prices that has occurred since their last pay rise. So the investigation into the question 'Are people getting better off?' will involve comparing *changes* in prices with *changes* in earnings.

A television journalist in the late 1980s summed up the relationship between pay claims and price rises in the following words: 'As people see prices go up in the shops, they'll continue to demand bigger pay rises, that in turn could feed through into inflation—a vicious circle that Mr Lawson has to try and break.' So while there is always pressure from employees for pay to keep ahead of prices, there is also pressure from government and employers to restrain pay rises and keep costs down.

Nigel Lawson was the UK Chancellor of the Exchequer at the time.

Factors other than inflation also play a part in determining the level of pay settlements in different occupations and companies and in different parts of the UK. One factor is how easy it is to find suitably skilled employees, and the scope for productivity gains is another. The size of the group of unemployed workers also has an effect. Clearly some groups of workers will obtain larger pay rises than others in any year. Some may obtain pay rises greater than the rate of inflation and some smaller. So some groups will be better off while others will not. The question 'Are people getting better off?' once again seems simplistic. However, changes in *average* earnings give some insight into whether, in general, (employed) people are getting better off. But, because this involves looking at averages, these conclusions cannot be valid for everyone.

In Section 6 of *Unit 2*, you saw how the Retail Prices Index is used to measure changes in the overall level of prices. An index of earnings, the *Average Earnings Index* (AEI), can be used to measure *changes* in earnings.

7.1 The Average Earnings Index

The Average Earnings Index (AEI) is calculated by the Department for Education and Employment once a month. It measures *changes* in employed people's main source of income: their earnings. The data used to calculate the AEI come from a survey called the *Monthly Return of Total Wages and Salaries.* Whereas the *New Earnings Survey* covers a sample of *individuals*, this survey covers a sample of *firms.* Each month, a sample of firms is sent a simple questionnaire.

If you are interested in finding out more about the AEI, it is discussed in greater detail in Block A of the OU course MDST242 *Statistics in Society.*

The information obtained from each firm includes the total number of employees and the total gross amount paid to these employees (including overtime, holiday pay, and other payments). No deductions are made for tax, national insurance and pension contributions. Fees paid to directors are excluded. The earnings of all employees, from the lowest-paid to the highest-paid, are added together to provide the total gross amount paid by the firm. For the Average Earnings Index, *earnings* simply means *gross amounts paid to employees.*

In order to understand what information the AEI provides and how to use it to measure changes in earnings, you need to know a little about how it is calculated. There is a great deal of similarity between the ways of calculating the AEI and the RPI. Recall that the goods and services whose prices are used to calculate the RPI are divided into groups, subgroups and sections. In a similar way, the firms and organizations covered by the AEI are classified according to the type of work they do: they are divided into twenty-five groups which are each subdivided into subgroups. Altogether there are about two hundred subgroups. This is illustrated in Figure 20.

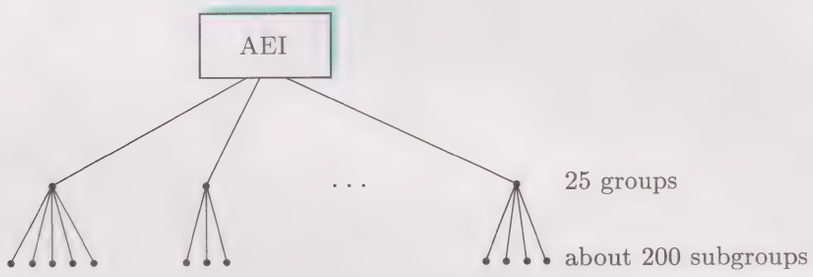


Figure 20 Levels of grouping of the AEI

The first stage in calculating the index is to find the *average weekly earnings* for each subgroup: this is simply the total gross amount paid to all employees of firms in the subgroup divided by the total number of employees of these firms. Next, the *average weekly earnings for each group* is found using a weighted mean. The *overall average weekly earnings* is a weighted mean of the average weekly earnings for the twenty-five groups.

The Average Earnings Index (AEI) is calculated by comparing the overall average weekly earnings with the corresponding figure for the base year. At the time of writing (1994), the base year is 1990: the average value of the AEI for 1990 is set at 100. Each month, the overall average weekly

earnings is compared with the overall average weekly earnings for 1990. Notice that, whereas for the RPI average prices are compared with those in the previous January, for the AEI average earnings are compared with those for the whole base year.

The AEI provides information on changes in the overall level of earnings in Great Britain. Recent figures for the index are published each month in the *Employment Gazette*.

Figure 21 shows the value of the Average Earnings Index for each month between January 1990 and December 1993. The plotted points have been joined by straight lines so that any patterns in the plot can be seen more clearly. (Notice that during 1990, the value of the index was less than 100 in some months and greater than 100 in others. The *average* value for 1990, the base year, was 100.)

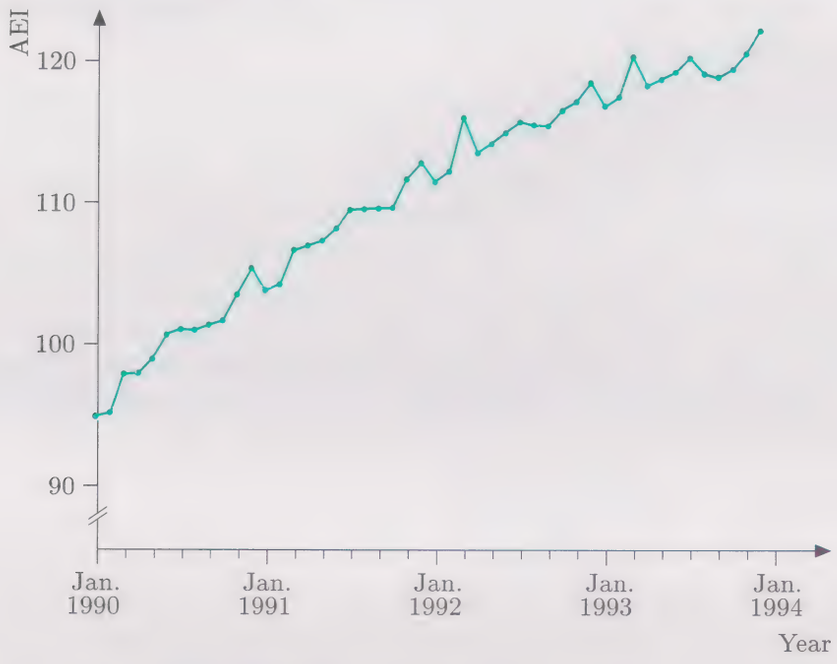


Figure 21 The Average Earnings Index, 1990–1993

Activity 38 *The peaks and dips in average earnings*

From Figure 21, it is clear that between January 1990 and December 1993 there was an upward trend in average earnings. However, average earnings did not rise steadily throughout this period: there are peaks and dips in the graph. Can you suggest any explanation for these patterns?

Some patterns are seasonal in nature; that is, they occur each year at the same time of the year. Because of the seasonal patterns in earnings, the index is only really useful for calculating changes in earnings from year to year. If it is used to calculate changes in earnings over periods of six

months or eighteen months, for instance, then the results may be misleading, unless seasonally adjusted.

Example 7 Using the AEI

Estimate the percentage increase in average earnings over the year.

The value of the AEI for October 1992 was 116.0 and the value for October 1993 was 118.4.

The value in October 1993 as a percentage of its October 1992 value was

$$\frac{118.4}{116.0} \times 100\% \simeq 102.1\%.$$

Thus, average earnings increased by 2.1% over the year from October 1992 to October 1993.

Activity 39 Calculating changes in average earnings

The value of the AEI for June 1992 was 114.5 and its value for June 1993 was 118.5. Find the percentage increase in the value of the AEI between June 1992 and June 1993.

7.2 Prices and earnings

The central question which motivated investigations of prices and earnings was: 'Are people getting better off?' At the beginning of *Unit 2*, you saw that this is a difficult question to answer precisely. There are many different factors to take into account and also what may be true for one person or household may not be true for another. However, two key factors, prices and earnings, are certainly important.

In *Unit 2*, you saw how the Retail Prices Index (RPI) is used to measure changes in prices; and you have just seen how to use the Average Earnings Index (AEI) to measure changes in earnings. In this subsection, the RPI will be used together with the AEI to compare changes in prices and earnings.

To compare changes in the RPI with changes in the AEI, changes in both indices must be calculated. For each month in the period from 1980 to 1993 the percentage increase in the value of the index over its value a year earlier was calculated for both the RPI and the AEI. The results are shown in Figure 22.

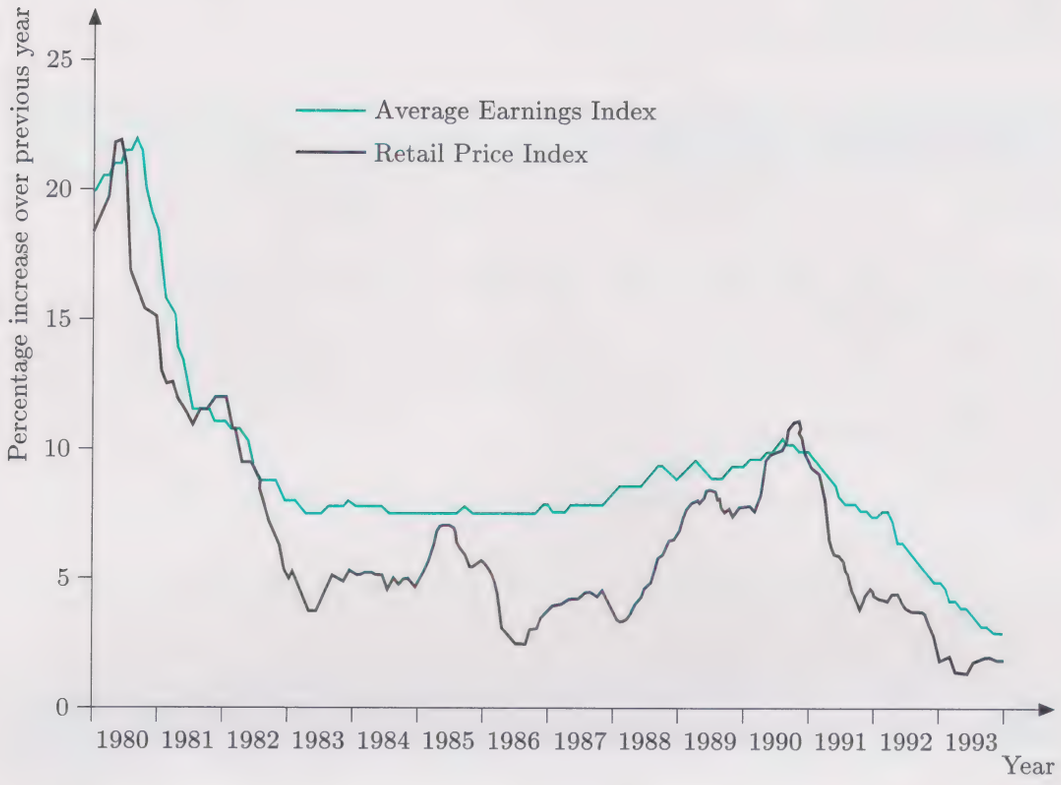


Figure 22 Changes in the AEI and the RPI: 1980–1993 (increases over previous year)

Activity 40 *Interpreting the graph*

What does the graph reveal about changes in prices and earnings during the period from 1980 to 1993?

First, remember that these are graphs of *increases* over a year earlier. Since the graphs remain above the horizontal axis (the level which corresponds to no change over a year earlier), both prices and earnings were rising throughout the whole period. At no time during the period were either prices or earnings lower than they were a year earlier. Even though the graphs appear to be falling in the early 1980s and early 1990s—the *rate* of inflation and the average level of pay *increases* were falling—both prices and average pay themselves were still *rising* throughout the whole period.

Figure 22 shows that, for the middle part of the period (1982–1989), the increase in the AEI remained steady at about 8% per year. Since the graph of the increase in the AEI is above the graph of the increase in the RPI between 1982 and 1989, the annual increase in average earnings was consistently higher than the annual rate of inflation: average earnings rose faster than prices. However, at the beginning and end of the period, there were occasions when the reverse was true: prices were rising more quickly

than average earnings. (This is shown by the fact that at times the graph of the annual increase in the RPI was above the graph of the annual increase in the AEI.)

Example 8 Comparing changes in prices and earnings

One way to compare changes in prices and earnings is to calculate the AEI ratio as a percentage of the RPI ratio.

(a) The value of the AEI for January 1992 was 111.1 and the value for January 1991 was 103.8.

Therefore, the value of the AEI in January 1992 as a percentage of its January 1991 value was:

$$\frac{111.1}{103.8} \times 100\% \simeq 107.0\%.$$

Thus, the increase in the value of the AEI over the year January 1991 to January 1992 was 7.0% of the January 1991 value.

(b) The values of the RPI in January 1992 and January 1991 were 135.6 and 130.2 respectively.

Thus, the value of the RPI in January 1992 as a percentage of its value a year earlier was:

$$\frac{135.6}{130.2} \times 100\% \simeq 104.1\%.$$

So average prices increased by 4.1% over the year.

(c) Now compare the changes in the two indices by calculating the ratio $\frac{107.0}{104.1}$: that is, $\frac{\text{AEI ratio}}{\text{RPI ratio}}$, and expressing this as a percentage:

$$\frac{107.0}{104.1} \times 100\% \simeq 102.8\%.$$

Hence earnings increased more than prices.

Comparisons like this give us a measure which is called the *real earnings for that month compared with one year earlier.* Real earnings for January 1992 were 102.8% of their value a year earlier. The term 'real earnings' is used here because the ratio of actual earnings (given by the AEI for that month divided by the AEI one year earlier, divided by the corresponding ratio for prices, obtained from RPI values) gives a measure of the change in the purchasing power of earnings. To calculate this measure directly the following formula is used.

This figure is often expressed as a percentage.

Real earnings for month A compared with one year earlier =

$$\frac{\text{AEI for month A}}{\text{AEI for month A, one year earlier}} \div \frac{\text{RPI for month A}}{\text{RPI for month A, one year earlier}}$$

You will not have to reproduce this reasoning, but you will need to be able to use both forms of the formula.

Dealing with real earnings

This formula has a complex structure, dividing one ratio by another. This structure can be expressed mathematically using symbols p, q, r and s as follows.

$$\frac{p}{q} \div \frac{r}{s} = \frac{p/q}{r/s}$$

where
$p =$ AEI for month A
$q =$ AEI for same month one year earlier
$r =$ RPI for month A
$s =$ RPI for same month one year earlier

If you multiply the top and the bottom of any fraction by the same number at the same time, there is no change in the *value* of the fraction as a whole. But there will be a change in the *form*.

If you multiply both the top and bottom of this fraction by the number $\frac{s}{r}$, it becomes:

$$\frac{\frac{p}{q} \times \frac{s}{r}}{\frac{r}{s} \times \frac{s}{r}}$$

Note: $\frac{s}{r}$ was chosen so that the bottom part of the fraction simplifies to 1, since $\frac{r}{s} \times \frac{s}{r} = 1$.

So a different way of writing the same fraction is

$$\frac{\frac{p}{q} \times \frac{s}{r}}{1} = \frac{p}{q} \times \frac{s}{r}$$

Translating, using the above letter equivalents, this says that an identical way of working out the real earnings for month A, compared with one year previously, is:

$$\frac{\text{AEI for month A}}{\text{AEI for month A, one year earlier}} \times \frac{\text{RPI for month A, one year earlier}}{\text{RPI for month A}}$$

This is demonstrated in Example 9.

The list of values of the AEI and the RPI in Table 18 will be used to illustrate this calculation.

Table 18 Values of the AEI and RPI in 1992 and 1993

	AEI		RPI	
	1992	1993	1992	1993
January	111.1	116.1	135.6	137.9
February	111.9	116.7	136.3	138.8
March	115.8	119.6	136.7	139.3
April	113.0	117.5	138.8	140.6
May	113.9	118.0	139.3	141.1
June	114.5	118.5	139.3	141.0
July	115.1	119.5	138.8	140.7
August	114.6	118.2	138.9	141.3
September	114.7	118.0	139.4	141.9
October	116.0	118.4	139.9	141.8
November	116.4	120.0	139.7	141.6
December	117.9	121.6	139.2	141.9

Example 9 *Using real earnings formulas*

Find the increase in real earnings for November 1993 compared with November 1992.

Real earnings for November 1993 compared with one year earlier:

$$\frac{\text{AEI for November 1993}}{\text{AEI for November 1992}} \div \frac{\text{RPI for November 1993}}{\text{RPI for November 1992}}$$

$$= \frac{\text{AEI for November 1993}}{\text{AEI for November 1992}} \times \frac{\text{RPI for November 1992}}{\text{RPI for November 1993}}$$

$$= \frac{120.0}{116.4} \times \frac{139.7}{141.6} \simeq 1.017.$$

To express this as a percentage, multiply by 100, which gives 101.7%.

So real earnings for November 1993 were 101.7% of their value a year earlier.

Activity 41 *Real earnings*

For each of the following months calculate (as a percentage) the real earnings for that month compared with one year earlier.

(a) March 1993 (b) June 1993 (c) September 1993

Table 19 shows the real earnings for each month in 1993 compared with one year earlier.

Table 19 Real earnings compared with one year earlier (as a percentage): 1993

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
102.8	102.4	101.4	102.7	102.3	102.2	102.4	101.4	101.1	100.7	101.7	101.2

So, for each month in 1993, real earnings compared with one year earlier were greater than 100%. According to this measure, employed people were getting better off during 1993.

7.3 So, are people better off?

The central question posed at the start of *Unit 2* was: 'Are people getting better off?' In attempting to answer this question initially, you were asked to consider a simple, everyday measure—the cost of a loaf of bread—and then asked what proportion of a daily wage it represented. However, this measure is clearly inadequate as it looks at only one of thousands of goods and services that people purchase and which affect how well-off they think they are. But the principle of looking at changes in prices compared with changes in earnings was one that was sustained throughout the two units. You have looked at more formal measures, the RPI and the AEI, which, respectively, monitor changes in prices from a large basket of goods and services, and changes in earnings from a wide range of different occupations and employers.

All the evidence from this section so far suggests that, on average, people in employment *have* been getting better off. For example, Figure 22 indicates that, since 1980, there were only occasional months when the annual rate of inflation was greater than the increase in the AEI over the previous year. For the vast majority of months, the annual increase in average earnings has more than offset the average annual price rise. Of course, this conclusion only applies to the employed; it does not necessarily follow that pensioners, the unemployed or the self-employed have also, on average, been getting better off.

This section ends with a brief look at three wider issues:

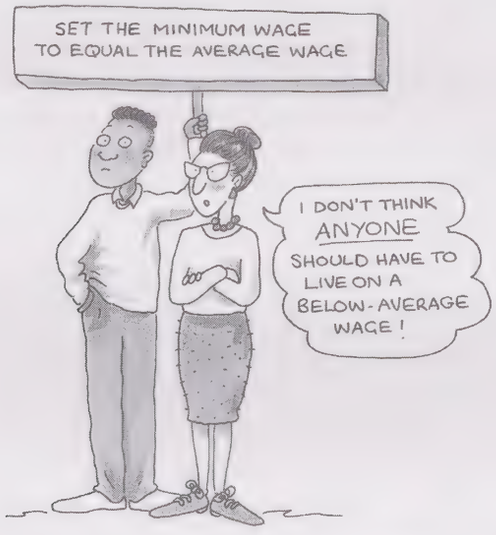
- ◇ the fact that averages are not individuals;
- ◇ the effect of tax;
- ◇ whether there is more to being better off than *material well-being.*

Averages are not individuals

It has already been stressed that both the AEI and the RPI are based on *averages* and should not be taken as representing the circumstances of any particular individual or group of individuals. So the AEI and the RPI, even when used together, provide only a very poor assessment of whether a particular group of people is getting better off.

The average being referred to here is the mean. Of course, unlike the median, the mean is often affected by extreme values. It only needs a small group of individuals to secure very large rises in their earnings to produce a substantial increase in the mean earnings. This tells you very little about how everyone else has fared.

It is important to stress, therefore, that the AEI is not sensitive to changes in the overall distribution of earnings—it is only concerned with *averages*, which here means the *mean*.



A further problem with the AEI can be illustrated by the following example.

Example 10 Mean earnings after redundancy

In Activity 8 you were introduced to the fictitious firm Troublefree Computers. The initial earnings of the five employees were £200, £250, £300, £350 and £400. So the mean earnings of the five employees was £300. Suppose that the lowest-paid employee is made redundant. What effect do you think this would have on the AEI?

The mean earnings for the four remaining employees becomes

$$\frac{250 + 300 + 350 + 400}{4} = £325.$$

The average pay of those remaining employed has gone up!

So, if poorly-paid workers are laid off, then the value of the average pay of those remaining will go up. This means that the AEI will increase! This may seem paradoxical, and is indeed difficult to square with the feeling that if the AEI is increasing then people are generally getting better off.

However, the paradox is somewhat explained when you consider that the AEI is based only on those in employment, so it can increase purely as a result of lower-paid workers being laid off and no longer being counted.

The effect of tax

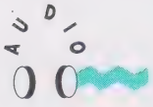
A further complication occurs when the effects of income tax are taken into account. A rise in gross earnings of, say, 4% will be unlikely to result in a corresponding rise in net earnings of 4%—it may be less or more than this figure. There are two reasons for this. First, income is taxed at different rates for different bands of income, so the effect of tax will vary depending on how much is earned. Second, each year the tax allowances tend to be adjusted and this also influences the proportion of earnings that are taxed. So, it is increases in net earnings which must be considered in relation to price changes, not increases in gross earnings.

Income tax also has, potentially, a role in the redistribution of wealth between rich and poor. In 1979, when there was a change in government from Labour to Conservative, the so-called marginal rate of tax for the wealthy (that is, the percentage paid in tax for the last £1 earned) was 83%. By the 1990s, this marginal rate had dropped to 40%.

Another factor to be considered is the balance between direct tax (basically another term for income tax) and indirect tax (for example, value added tax (VAT) charged by the UK government on goods and services). Compared with the higher-paid, the lower-paid need to spend a high proportion of their earnings. Thus, a larger proportion of their earnings is paid to the government in indirect tax. Taking direct tax and indirect tax together, the top 10% of households pay about 32% of their earnings in tax, while the bottom 10% pay around 43%.

Is there more to being better off than material well-being?

This section ends with an audiotape of a group of pensioners talking about whether they feel better off. This is an opportunity to broaden the debate from material (and numerical) measures to look at other factors.



Activity 42 *Better off than before?*

Before listening to the audiotape, note down some other factors which you consider to be important measures of how well-off (or how badly-off) you feel yourself to be.

Now listen to band 1 of Audiotape 2. Make a note of some of the factors that were offered by those who spoke.

Here are some questions for you to think about.

- ◇ How might some of these factors be measured?
- ◇ Did the speakers' factors differ from the items on your list? Why?
- ◇ Do you feel there is a consensus on what factors are important as measures of how well-off (or how badly-off) people feel?

A clear message of this final subsection has been that making judgements about people's earnings based solely on averages like the AEI can be very misleading. Indeed, the same warnings need to be applied to all average measures of earnings and price changes as they say nothing about the inequalities experienced by individuals. Taken overall, average earnings increases do seem to have exceeded price rises during the 1980s and early 1990s and there seems to be good evidence for claiming that, *if you are still in work*, you are probably better off. However, as inequalities have widened over the last ten years, a higher proportion of people have become either low-paid or unemployed. So, while the rich have got richer, relatively speaking, the poor have got poorer.

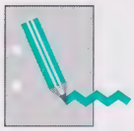
Outcomes

After studying this section, you should be able to:

- ◇ use the AEI to calculate the annual increase in average earnings (Activity 39);
- ◇ use the AEI and the RPI to calculate the real earnings at one date compared with an earlier date (Activity 41);
- ◇ interpret a graph of the AEI or of annual increases in the RPI and the AEI (Activities 38 and 40);
- ◇ identify factors other than material ones which affect how well-off people feel (Activity 42).

Unit summary and outcomes

You now know something about how UK government statisticians measure changes in prices and earnings. So you should be able to explain what politicians and journalists really mean when they make broad claims about whether or not 'we' are getting better off. In the course of discussing earnings, several mathematical ideas have been introduced and used—for example, ratios and index numbers, quartiles and deciles, range and interquartile range, boxplots, earnings ratios, and real earnings.



Activity 43 *Looking back*

Now would be a good time to reflect for a few minutes on your progress over these two units as a pair. Think about what you knew at the beginning of the units and compare it with what you know now. Ask yourself whether there are any topics which you want to summarize and add to your Learning File, along the lines of the entry you wrote on weighted means in *Unit 2*. If so, perhaps do it now.

Also take time to reflect on the progress you have made in learning mathematics. Which topics in this unit have you found straightforward? Which have you found difficult? Write down what you feel you have gained from studying this unit—for example, a skill that you have improved or an understanding of some idea or technique.

Write down one example of something that caused you difficulty and on which you feel you need to spend more time. If you have identified some aspect of the work in this unit that is causing you real concern, how are you going to go about overcoming this? Several possibilities were suggested at the end of *Unit 2* (the preparatory materials, other students, your tutor). If you identified something in *Unit 2* that required action on your part, what did you try then? What was the result of your action? (Did you remember to record the results of your action in your Learning File?) Use your experience from that occasion to help you to decide what to do now.

There is printed printed response sheet for this activity.

Outcomes

You should now be able to:

- ◇ specify the types of data needed to investigate claims about earnings;
- ◇ extract relevant information from tables of data;
- ◇ explain the meanings of the terms 'range', 'lower quartile', 'upper quartile', 'interquartile range', 'lowest decile' and 'highest decile';
- ◇ comment on the features of different diagrams (for example, W-shaped diagram, boxplot) offered to support your thinking;
- ◇ find the range, the lower quartile, the upper quartile and the interquartile range for a given batch of data;
- ◇ draw a rough sketch and an accurate version of a boxplot for a given batch of data with the aid of your calculator;
- ◇ use your calculator to draw boxplots and calculate various summary statistics (such as quartiles) from batches of data;
- ◇ draw decile boxplots to represent earnings data given in the form of summary statistics;
- ◇ make comparisons between distributions using boxplots and decile boxplots;
- ◇ explain the meaning of the term 'earnings ratio' and calculate and interpret earnings ratios;
- ◇ extract, interpret and criticize statistical information from appropriate articles on earnings;
- ◇ explain how the RPI (Retail Prices Index) and the AEI (Average Earnings Index) may be used to offer a partial resolution to the question 'Are people getting better off?' and discuss the limitations of any conclusions;
- ◇ identify a range of factors which affect how well-off people feel.

Appendix: Drawing accurate boxplots

Introduction

You have seen how to obtain a boxplot for a batch of data on the screen of your calculator and how to use this to help you to draw a rough sketch of the boxplot. This appendix provides a procedure for drawing accurate boxplots by hand on squared or graph paper.

Drawing an accurate boxplot

This subsection works through the drawing of an accurate boxplot based on the earnings of the eleven male solicitors from Table 12 (page 35). The data will be used to illustrate choosing a scale and drawing a boxplot. If you are not happy about drawing boxplots, follow this example through, drawing the boxplot yourself.

Here is a rough sketch of the boxplot for this batch of data as a guide.

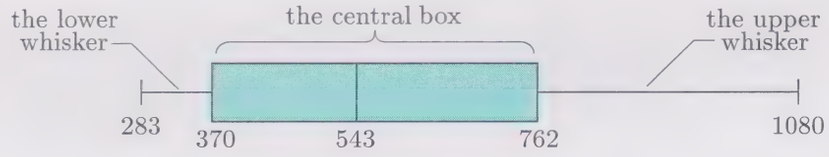


Figure 23 A rough sketch of a boxplot for the earnings of eleven male solicitors

One of the advantages of a boxplot is, unlike a conventional graph, you do not have to read points off it: the summary values are written on it (so are stressed) and the remaining data values are not (so are ignored).

Thinking about a suitable scale is the essential first step in drawing a boxplot. There are several general points to remember that will help you to make a sensible choice.

- ◇ All the data must fit on the page: the whole range of values in the batch of data, from the lower extreme *min* to the upper extreme *max*.
- ◇ Choose round numbers as endpoints of your scale; for example, 100 and 500, rather than 107 and 474. This means the lines on the graph paper will mark convenient numbers.
- ◇ The boxplot must fit on the paper, but you must be able to see clearly all the important features of the batch of data (so not too small).
- ◇ Make one square on your paper correspond to a convenient whole number such as 2, 5 or 10 (or 20 or 50, etc.), rather than 3 or 7 or 1.8. This will make it easier to find points on the scale.

Choosing the endpoints

Moving out beyond the endpoints to suitable 'round' numbers suggests a scale with endpoints of £200 and £1100, or even £0 and £1200.

Choosing the scale

You now need to work out how to fit the boxplot conveniently and clearly on your paper. The scale you choose will depend on the size of the paper you use, but, for this example, suppose that you are using A4 paper. A sheet of A4 paper is approximately 21 cm wide so, as a very rough guide, try to make the useful part of your scale about half to three-quarters of the width of the paper—in the region of 10 cm to 15 cm long. For the male solicitors' earnings data, your scale needs to go from £0 to £1200, so it must cover a range of £1200. If you make this correspond to 12 cm, then £100 will correspond to 1 cm.

Drawing the scale

You are now in a position to draw the line on which the scale will be marked—called the *axis*. First, decide where to put your line. Leave some space above the line in which to draw the boxplot: 3 cm should be sufficient. Now draw a horizontal line across the page, longer than the length you need to use (in this case, longer than 12 cm). Larger values will be towards the right-hand end of the axis, indicated by an arrowhead as shown below. To show the scale, mark the axis with vertical lines at suitable regular intervals; in this case, every 2 cm or £200. For the male solicitors' earnings data, the range of £1200 is to correspond to 12 cm, so if you mark points at 2 cm intervals from 0 to 1200 then you will have seven marked points as shown in Figure 24.

Figure 24 Draw the axis

Finally, label the axis with numbers, and write the quantity measured (earnings) and the units (£) in which the values are measured below the arrow. Although £100 corresponds to 1 cm, you do not need to mark every point. You could mark intervals of £200 or £500.

Figure 25 Label the axis

At this point, pause to ask yourself the following question: is the scale easy to use? Will the intervals make for easy plotting? That is, can you find points easily on the scale? (For example, can you find the point corresponding to £480 easily?) If the scale is unsatisfactory, go back and choose a more suitable scale.

The box and whiskers

Having decided on and drawn the scale, it only remains to draw the box and whiskers and label the key points on the boxplot as in your rough sketch.

First, draw the box about 1 cm above your axis. Nothing is represented by the *thickness* of the box. Your diagram will look better if the box is neither very thin nor very fat. Draw vertical lines in line with the positions on the scale of the lower and upper quartiles (370 and 762 in this example). Complete the box by joining the two vertical lines. Mark the median (543 in this instance) by drawing a vertical line through the box in line with the position of the median on the scale, as shown in Figure 26.

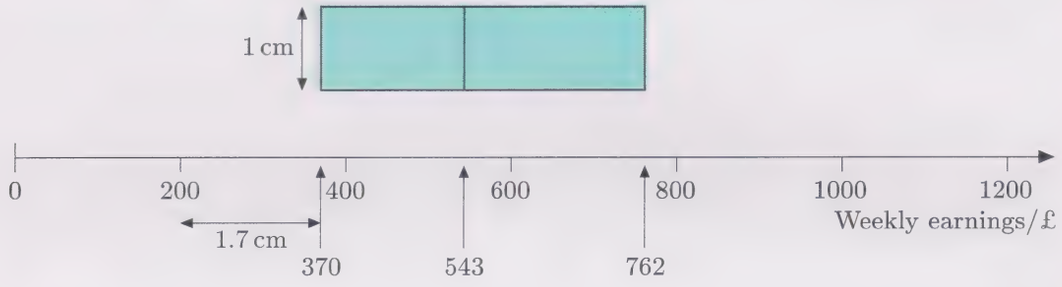


Figure 26 Draw the box

Mark the two extremes, *min* and *max*, by two short vertical lines in line with the positions of *min* and *max* on the scale (283 and 1080 in this instance). Position them so that each can be joined half-way up the box, as shown in Figure 27.

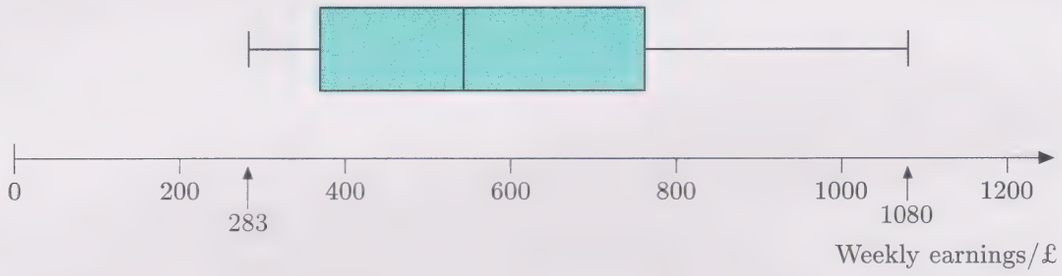


Figure 27 Draw the whiskers

Complete the boxplot, by transferring the five values from the rough sketch, as shown in Figure 28.

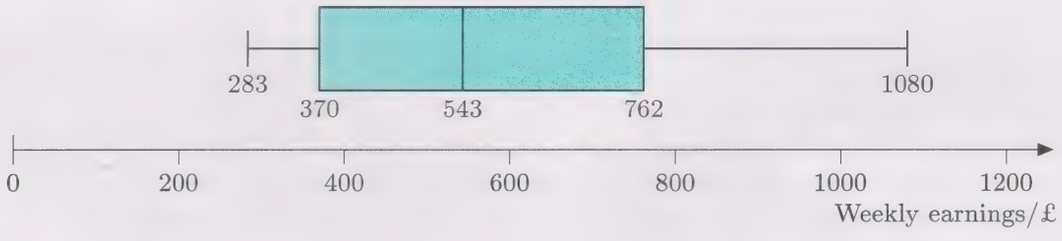


Figure 28 Label the key values

When you have drawn your boxplot, compare it with the boxplot on the screen of your calculator to check that they are the same shape and that you have not made any errors.

You can also draw two or more boxplots using the same scale, as a useful way of comparing two batches of data. If you are drawing two boxplots on the same scale, then it may be easier to draw one above the scale and the other below the scale. But you may find it easier to compare them if they are next to each other. Make sure you draw the boxes the same width, so that your diagram does not give a misleading impression. Label both boxplots clearly to distinguish between them. This is shown in Figure 29 for the male and female solicitors' earnings (using the data from Table 12).

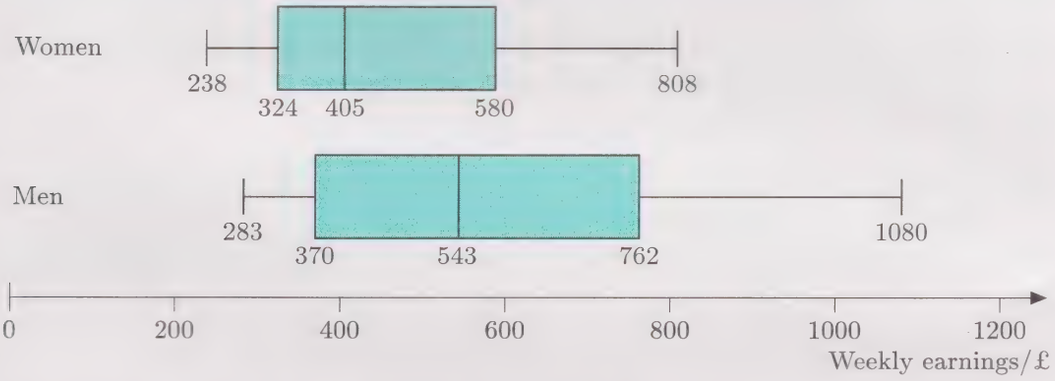


Figure 29 Boxplots of gross weekly earnings of solicitors

Activity 44 *Drawing accurate boxplots*

Sketches of the boxplots of the chefs' and cooks' earnings data of Table 13 (page 39) are shown in Figure 17 (page 45). Choose a suitable scale and then draw the axis and accurate boxplots.

Table 20 contains earnings data for a small number of male and female nurses, and for male and female secondary teachers. In the next two activities, you are asked to use these batches of data to compare the weekly earnings of men and women in these occupations. This is to provide you with practice at using your calculator to obtain boxplots, drawing rough sketches of and accurate diagrams of boxplots, and interpreting the information contained in them.

Table 20 Weekly earnings (in pounds)

Nurses											
Women	266	310	420	156	287	372	235	202	363	297	
Men	355	160	389	321	252	225	306	340	463	408	285

Secondary teachers										
Women	378	436	502	270	385	404	340	454	390	
Men	441	460	366	652	390	313	481	415	538	455

Activity 45 *Drawing and interpreting boxplots*

(a) Use your calculator to obtain boxplots of the nurses' earnings data in Table 20, and hence draw a rough sketch of the boxplots.

(b) Draw your own accurate diagram of the boxplots.

(c) What do your boxplots tell you about the relative earnings of male and female nurses?

Activity 46 *More drawing and interpreting boxplots*

(a) With the aid of your calculator, sketch boxplots for the earnings data in Table 20 for male and female secondary teachers.

(b) Draw your own accurate diagram of the boxplots.

(c) What do your boxplots tell you about the relative earnings of male and female secondary teachers?

Comments on Activities

Activity 1

See the comments after the activity.

Activity 2

One possible explanation is that (even in the 1990s) many women do not work while their children are young; far fewer men than women stay at home to look after young children. This means that very young female workers, those not yet on adult rates, form a greater proportion of all female workers than is the case for male workers. Also (in the 1990s) in the UK women tend to retire at a younger age than men, so even if both women and men were in employment throughout their 'working lives', the proportion of workers on adult rates would be higher for men than for women.

Activity 3

See the comments after the activity.

Activity 4

(a) Women worked 37.4 hours per week on average—fewer hours than the average worked by men, which was 41.3 hours.

(b) On average, men did 2.3 hours more overtime per week than women ($3.1 - 0.8 = 2.3$). Alternatively, men do nearly four times as much overtime as women ($3.1/0.8 = 3.875$).

(c) Removing overtime pay from the gross weekly earnings figures would reduce the men's figure more than the women's figure, since men do more overtime. You would expect this to narrow the 'gap' between men's earnings and women's earnings and therefore *increase* the earnings ratio.

(d) Since men work more hours per week on average than women, you would expect men's gross weekly earnings to be more than women's, even if they were paid the same for similar amounts of work. A fairer comparison might be to look at the gross *hourly* earnings of men and women. This would eliminate the effect on earnings of men working more hours per week than women.

Activity 5

(a) The earnings ratio at the mean based on weekly earnings excluding overtime is

$$\frac{247}{330} \simeq 0.7485 \text{ or approximately } 75\%.$$

(b) The earnings ratio at the mean based on hourly earnings is

$$\frac{668}{847} \simeq 0.7887 \text{ or approximately } 79\%.$$

(c) See the comments after the activity.

Activity 6

(a) The earnings ratio at the mean for each year is given in the table below.

Year	1979	1981	1983	1985	1986	1987
Earnings ratio (%)	73	75	74	74	74	74

Year	1988	1989	1990	1991	1992	1993
Earnings ratio (%)	75	76	77	78	79	79

(b) There was very little change in the earnings ratio during the 1980s, although there was a slow, steady increase between 1987 and 1992.

(c) Since the earnings ratio has increased, this measure suggests gender inequalities in earnings have narrowed.

Activity 7

(a) The earnings ratios at the median are given in the table below.

Earnings ratios at the median	(%)
Gross weekly earnings including overtime	73
Gross weekly earnings excluding overtime	78
Gross hourly earnings excluding overtime	81

(b) As was the case when using the mean, the earnings ratio at the median increases when overtime is excluded and again when hourly earnings are considered instead of weekly earnings. In each case, the earnings ratio at the median is higher than the corresponding earnings ratio at the mean.

Activity 8

(a) The mean earnings and the median earnings required in parts (a) and (b) are given in the table below, together with the mean and median for several other possible values of the manager's earnings.

Manager's earnings	Mean	Median
400	300	300
500	320	300
600	340	300
700	360	300
800	380	300
1000	420	300
5000	1220	300

(b) See part (a).

(c) The median is unaffected by the increases in the manager's earnings, whereas the more the manager's earnings increase, the greater the mean becomes.

Activity 9

(a) 50% of people earn less than the median wage, so a person of median earnings will pass by halfway through the parade at 10.30 am.

(b) We are told that a person of mean height passes by twelve minutes before the end of the parade—that is, at 10.48 am. Since 10.48 am is 48/60 or $\frac{4}{5}$ of the way through the hour, $\frac{4}{5}$ or 80% of people earn less than the mean wage.

(c) Some images can be misleading!

In the cartoon, a person earning twice the average wage is drawn twice as tall as a person earning the average wage. However, that person is also drawn twice as wide—the tall people are not tall and thin—so the *area* of the cartoon taken up by a person earning twice the average wage is *four* (2×2) times the area taken up by a person earning the average wage. And, in practice, a reader may well interpret a person in the cartoon as a figure occupying a *volume* in space. So the impression received is of a figure *eight* ($2 \times 2 \times 2$) times as large. Thus, the effect of the cartoon is to exaggerate the differences in earnings of different people.

Unfortunately, many published diagrams make use of area or volume to exaggerate the visual effect of points they are trying to make: look out for this whenever you read an article or see a television programme where diagrams are used to support the arguments.

In addition, the cartoon is based on the idea that greater height corresponds to greater income. This choice has quite strong psychological overtones to do with cultural norms of 'stature', 'importance', and so on: it is a far from neutral image. Consider the impact of a redrawn cartoon where the key image was a person with their hand outstretched: the larger the salary, the longer the arm.

One reason you may be able to orientate yourself with regard to the cartoon as it stands is that you have plenty of experience of the distribution of people's height to bring to bear on interpreting this image. Most importantly, there is no scale, other than the notion of 'average height'.

Activity 10

To investigate whether or not women are being paid less than men for similar work, data are needed on the earnings of men and women in individual occupations. Ideally, these data would exclude overtime and, since men tend to work longer hours than women, hourly earnings would be desirable. To find out whether women are employed predominantly in occupations with low pay, you need to know how many men and women are employed in the different occupational groups.

Activity 11

See the comments after the activity.

Activity 12

The percentages of men and women in manual and non-manual occupations are given in the table below.

	Women	Men
Manual	18	45
Non-manual	82	55
	100	100

Whereas 45% of men are in manual occupations, only 18% of women are in occupations classified as manual. A far greater proportion of women than men are in non-manual occupations. So the patterns of employment of men and women are very different.

One effect of tables is to allow the two comparisons (men vs. women, manual vs. non-manual) to be made more directly: the eye moves easily from left to right and up and down. In particular, the percentage totals can be used to check your arithmetic.

Activity 13

See the comments after the activity.

Activity 14

The earnings ratio at the median for chefs and cooks is:

$$\frac{418}{478} \simeq 0.8745 \text{ or approximately } 87\%.$$

The earnings ratio at the median for cleaners and domestics is:

$$\frac{394}{427} \simeq 0.9227 \text{ or approximately } 92\%.$$

Using hourly earnings instead of weekly earnings has increased the earnings ratio in each case.

Activity 15

We chose to look at *even* hours of overtime only, to reduce the amount of calculation, while still looking systematically across most of the range.

The total earnings and hourly rates are given in the table below.

Total hours worked	40	42	44	46	48
Total earnings in £	160	172	184	196	208
Mean hourly rate in £	4.00	4.10	4.18	4.26	4.33

As you can see, the more overtime Ivor works, the higher his overall hourly rate becomes.

Activity 16

One possible explanation is that proportionately more women than men in each occupational group are in junior, lower-paid posts and proportionately fewer women than men are in senior, more highly-paid positions.

Activity 17

For the women's batch,

$$\text{range} = max - min = 808 - 238 = 570.$$

For the men's batch,

$$\text{range} = max - min = 1080 - 283 = 797.$$

The range of the earnings is greater for the men's batch than for the women's batch.

Activity 18

The main weakness of all three measures is that they do not take any account of the size of a batch of data. Consider proposed measure (b), for instance. For a small batch, excluding five values at each end may involve excluding all, or nearly all, of the values in the batch. For a large batch, excluding only five values may not exclude all the very unusual values. So the measure still may not accurately represent the spread of the main body of the data. Similarly, if only one value is excluded, as with measure (a), then an unusually large or small value may still remain. And using measure (c) for a small batch may mean that all the values are included, whereas using it for a large batch nearly all the values are excluded.

A balance is needed between excluding so much data that the measure is not representative of the batch of data and excluding so few values that the results are still strongly influenced by a few extreme values.

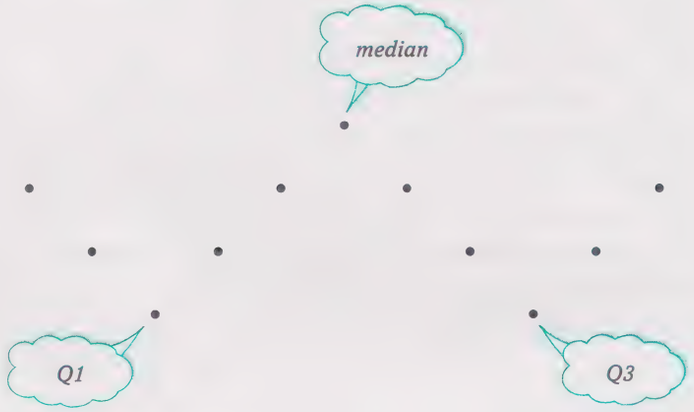
Activity 19

(a) For a batch of nine values, the configuration looks like the figure below.

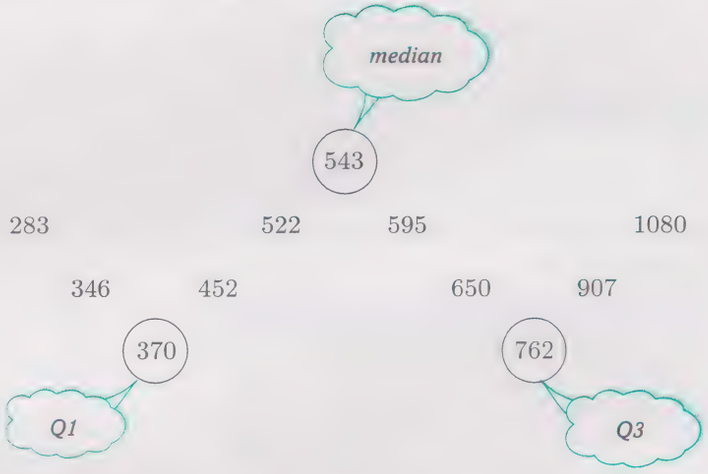
(b) A batch of ten values produces the following diagram.

Activity 20

(a) The diagram for eleven values looks like the following.



(b) Men's earnings



The diagram acts as a data template and shows which are the key data values. Matching the numerical data with the image, gives the following values:

median = £543,
$Q1$ = £370,
$Q3$ = £762.

(c) For the batch of women's earnings:

median = £405,
$Q1$ = £324,
$Q3$ = £580.

Activity 21

These comments include only some features of the W-shaped diagram.

The W image offers a way of looking at ordered numerical data. The letter W has five key points

and these are matched in turn with the values *min*, *Q1*, median, *Q3*, *max*. Any batch of data must have these five values, and so can always be shaped as a W.

The diagram stresses the idea of the data batch being ordered from left to right as your eye moves through the W.

It ignores the fact that a very familiar image for increasing numbers is a horizontal number-line. It is as if part of the number-line has been bent into a W-shape. But the uniformity of the W might wrongly suggest equal spacing between the five values: any sense of *scale* along the W has been suppressed.

A W-shape is made up of lines, whereas the use of the W-shaped diagram with batches of small size was with discrete blobs.

A blob represented a numerical value (here it is associated with a data point, which is a person's earnings).

Activity 22

The interquartile range for the batch of women's earnings is

$$Q3 - Q1 = £580 - £324 = £256.$$

This is much smaller than the interquartile range for the batch of men's earnings (£392). The earnings of the female solicitors are much less widely spread than the earnings of the male solicitors.

Activity 23

(a) The quartiles and extremes are given below.

	min	*Q1*	*Q3*	*max*
Women	110	132	191	235
Men	130	156	262	330

Female chefs and cooks:

range $= £235 - £110 = £125$;

interquartile range $= £191 - £132 = £59$.

Male chefs and cooks:

range $= £330 - £130 = £200$;

interquartile range $= £262 - £156 = £106$.

(b) The range of earnings is greater for the men than for the women. The interquartile range is also greater for the men than for the women, indicating that the greater spread for male chefs and cooks suggested by the range is not solely due to one or more men earning an unusually large amount.

Activity 24

From Figure 15, it appears that, since the upper quartile for the men is only a little less than the maximum for the women, nearly 25% of the men earned more than the maximum earned by any of the women. Since the box and right whisker are longer in the men's boxplot than in the women's boxplot, the earnings of the top 75% of earners are more widely spread for the men than for the women. Also *all* the key points on the boxplot—the median, the quartiles and the extremes—are higher for the men than the corresponding values for the women.

This shows that the level of earnings for the men is generally higher than for the women. This direct reading off of comparative statements from the two boxplots drawn on the same diagram with the same scale gives evidence that it is easier to compare the batches of data using boxplots than just by looking at the lists of the individual values in the batches.

Activity 25

(a) The earnings of female chefs and cooks are generally lower than the earnings of male chefs and cooks. For example, the maximum value for the women is lower than the upper quartile for the men, so more than 25% of the men earned more than the highest-paid woman. All five values which are marked on the boxplot are lower for the women than the corresponding values for the men. The box is longer for the men than for the women; the whiskers are also longer for the men. So there is a greater spread in the earnings of the men than in the earnings of the women (both the range and the interquartile range are larger for the men).

(b) Both boxplots are right-skewed: for each boxplot, the right-hand whisker is longer than the left-hand whisker and the right-hand section of the box is longer than the left-hand section. So both earnings distributions are right-skewed.

(c) Two boxplots of related batches of data drawn to the same scale make for very direct comparison: all you need to do is look up and down at any point. Because boxplots have a uniform structure in what they stress (quartiles, median, extremes) and what they ignore (most actual data values), looking at the key features makes a simple comparison of two batches easy.

Activity 26

(a) 50% of the men earned between £365 (the lower quartile) and £760 (the upper quartile).

(b) 25% of the women earned £329 or less, so £329 is the largest amount earned by the lowest-paid 25% of women.

(c) Approximately 25% of the men earned more than £760; and approximately 75% of the women earned less than £588.

Activity 27

(a) The highest decile for the female solicitors is £750, so 10% of the group earned £750 or more.

(b) £253 is the lowest decile, so 10% earned £253 or less.

(c) £399 is the median earnings and £750 is the highest decile. 40% of the women (50% − 10%: see Figure 18) received more than the median earnings but less than the highest decile, so 40% of the women earned between £399 and £750.

(d) 25% − 10% = 15% of the women earned between £253 (the lowest decile) and £329 (the lower quartile).

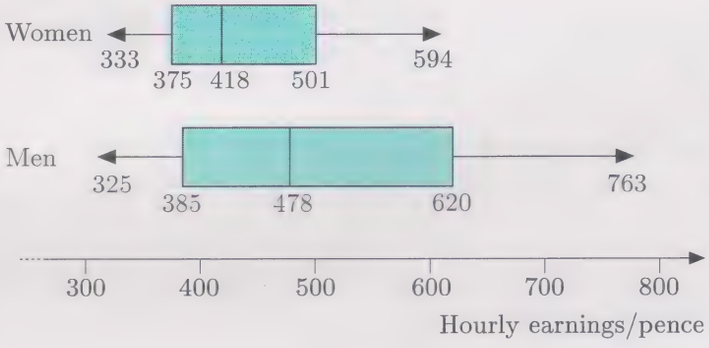
Activity 28

The spread of earnings is greater for men than women within both the box and the whiskers.

The lowest-paid men are paid more than the lowest-paid women; the median earnings figure is greater for the men than for the women; and the highest-paid men are paid more than the highest-paid women.

Activity 29

The decile boxplots below represent the gross hourly earnings of chefs and cooks.



The spread of the earnings was greater for the men than for the women, both within the box and in the whiskers. The earnings of the lowest-paid 25% of the men and women were very similar, but the median earnings figure was higher for the men, and the highest-paid 25% of the men earned more than the highest decile earnings of the women.

Activity 30

The earnings ratios are given in the table below.

Table 21 Earnings ratios for chefs and cooks

Highest decile	78%
Upper quartile	81%
Median	87%
Lower quartile	97%
Lowest decile	102%

These figures show that, at the bottom end of the earnings range, male and female chefs and cooks receive approximately equal pay for an

hour's work: the earnings ratios at the lowest decile and the lower quartile are 102% and 97% respectively. However, higher-paid women fare less well: the earnings ratios decrease steadily from the bottom of the distribution to the top. At the median, the earnings ratio is 87% and at the highest decile it is only 78%.

Activity 31

68% of £316.90 $\simeq$ £215.49
68% of £299.80 $\simeq$ £203.86

The mean gross weekly earnings *including* overtime has been used.

Activity 32

The definition of the decency threshold states that the mean gross weekly earnings is based on a thirty-seven and a half hour week. The average number of hours worked per week by full-time workers was approximately thirty-seven and a half in 1993 *excluding* overtime. Therefore, it would seem appropriate to use the mean gross weekly earnings *excluding* overtime to calculate the decency threshold. However, as you saw in Activity 31, the mean gross weekly earnings *including* overtime has been used.

Now consider the hourly rate quoted in Table 1 in the article. The mean gross weekly earnings including overtime (that is, for a forty-hour week) has been divided by 37.5 to obtain an hourly rate. £5.75 is 68% of this hourly rate. So the figure of £5.75 an hour is certainly questionable. Also, since the decency threshold has been calculated based on earnings *including* overtime, it would seem sensible to count the number of people whose earnings *including* overtime were less than this threshold. However, in Table 1, it states clearly that overtime earnings have been excluded when counting the number of full-time employees earning less than the threshold. This would have the effect of inflating the numbers of those with earnings below the threshold.

Activity 33

(a) The total number of people with gross weekly earnings below the decency threshold rose between 1988 and 1992, then fell in 1993. The figures were much lower in 1979 and 1982, but these figures are not directly comparable with those for later years (see the note in Table 1), so it is not possible to draw any conclusions from this.

(b) The percentage of men with earnings below the decency threshold has increased, whereas the corresponding percentage of women has decreased, so it would appear that men have fared worse than women. However, the percentage of women with earnings below the decency threshold has been much higher than the corresponding percentage of men throughout the period; this would suggest that women have fared worse overall.

(c) There are considerably more men than women working full-time, so 2.72 million is a greater percentage of the number of women working full-time than 2.73 million is of the number of men working full-time.

Activity 34

(a) In 1993, the lowest decile was approximately:

$$\frac{63.5}{100} \times £257.58 \simeq £163.56.$$

The highest decile was approximately:

$$\frac{157.6}{100} \times £257.58 \simeq £405.95.$$

(b) The earnings of the highest-paid have been rising at a faster rate than the median earnings.

Activity 35

(a) Since women tend to work less overtime and a shorter basic working week, excluding overtime and using hourly earnings instead of weekly earnings increases the earnings ratio. (Recall the discussion in Section 2.)

(b) You saw in Section 2 that the earnings ratio at the mean is generally lower than the earnings ratio at the median, so it does matter whether the mean or the median is used. Women are found to be doing worse if the mean is used rather than if the median is used. In fact, the mean has been used in Tables 6 and 7, thus representing the situation of women workers regarding pay as being even worse than it would have been had the median been used.

Activity 36

(a) The proportion of men with earnings below the decency threshold has doubled (14.6% in 1979, 29.3% in 1993), whereas the corresponding proportion of women has fallen (from 57.6% to 50.5%); see Table 1. Also, the earnings ratios at the mean (in Table 6 and Table 7) have risen between 1979 and 1993. These both suggest that the gap between men's and women's pay is closing. The only evidence the article offers to support the claim that the gap remains largely untouched is that between 1992 and 1993, there was no change in the earnings ratio based on hourly earnings excluding overtime (see Table 7). The author has ignored the steady closing of the gap between 1979 and 1992.

(b) How does the author know that progress towards equality has ended? No evidence is offered. Just because one figure did not change between 1992 and 1993, it does not mean that there will be no progress towards equality of pay in the future.

Activity 37

(a) First note that, in this context, a drop of 11.9 'percentage points' means a drop of 11.9% in the adult 'average', *not* a drop of 11.9% in the earnings of eighteen- to twenty-year-olds. Also, the increasingly common use of the phrase 'percentage point' rather than percent has the effect of turning '%' into a unit, rather than a certain percentage *of an amount*. Note, a drop of 10% in some quantity brings 60% to 54%, while a drop of 10 percentage points brings 60% to 50%.

(b) To check the claim, you need to know the 'average' gross weekly earnings for adults in 1993.

(c) The figure required (assuming overtime is being included in both figures used in the comparison) is the mean gross weekly earnings for all full-time workers in the UK given in Table 4: £316.90. (This figure includes overtime.)

(d) $$11.9\% \text{ of } £316.90 = \frac{11.9}{100} \times £316.90$$
$$\simeq £37.71.$$

Look again at the appropriate paragraph. It reads '$\cdots$ has slumped from 60.8 in 1979 to 49.9 per cent in 1993, a drop of 11.9 percentage points.'
But $60.8 - 49.9 = 10.9$, and

$$10.9\% \text{ of } £316.90 = \frac{10.9}{100} \times £316.90$$
$$\simeq £34.54.$$

Perhaps this was a printing error.

Activity 38

There are a number of patterns in the graph in Figure 21 that you may have commented on. Perhaps the clearest feature is the peak in the value of the AEI that occurs each December, which is followed immediately by a drop in January. The peak might possibly be due to extra payments leading up to the Christmas holiday period (overtime, holiday pay, etc.).

There is also a sharp rise each year in March. This might be due to there being more work (and therefore possibly more overtime) available in outdoor occupations as the weather improves in the spring. Or maybe it is due to bonus payments at the end of the financial year, or to a mixture of these and other factors.

There also appears to be a small peak each year in July: might this be due to holiday pay being

included in the survey returns? Another point to bear in mind is that many groups of workers receive a pay rise annually: if more of these rises occur in some months than in others, this could help to explain why the AEI increases more sharply at some times of the year than at others. You may well have spotted other patterns in the graph and suggested other explanations.

Activity 39

The value of the AEI in June 1993 as a percentage of its value in June 1992 was

$$\frac{118.5}{114.5} \times 100\% \simeq 103.5.\%$$

So the AEI increased by 3.5% of its June 1992 value between June 1992 and June 1993.

Activity 40

See the comments after the activity.

Activity 41

(a) The real earnings for March 1993 compared with one year earlier were calculated from:

$$\frac{\text{AEI, Mar. 1993}}{\text{AEI, Mar. 1992}} \div \frac{\text{RPI, Mar. 1993}}{\text{RPI, Mar. 1992}} \times 100\%$$

$$= \frac{\text{AEI, Mar. 1993}}{\text{AEI, Mar. 1992}} \times \frac{\text{RPI, Mar. 1992}}{\text{RPI, Mar. 1993}} \times 100\%$$

$$= \frac{119.6}{115.8} \times \frac{136.7}{139.3} \times 100\%$$

$$\simeq 101.4\%.$$

(b) The real earnings for June 1993 compared with one year earlier were calculated from:

$$\frac{\text{AEI, June 1993}}{\text{AEI, June 1992}} \div \frac{\text{RPI, June 1993}}{\text{RPI, June 1992}} \times 100\%$$

$$= \frac{\text{AEI, June 1993}}{\text{AEI, June 1992}} \times \frac{\text{RPI, June 1992}}{\text{RPI, June 1993}} \times 100\%$$

$$= \frac{118.5}{114.5} \times \frac{139.3}{141.0} \times 100\%$$

$$\simeq 102.2\%.$$

(c) The real earnings for September 1993 compared with one year earlier were calculated from:

$$\frac{\text{AEI, Sept. 1993}}{\text{AEI, Sept. 1992}} \div \frac{\text{RPI, Sept. 1993}}{\text{RPI, Sept. 1992}} \times 100\%$$

$$= \frac{\text{AEI, Sept. 1993}}{\text{AEI, Sept. 1992}} \times \frac{\text{RPI, Sept. 1992}}{\text{RPI, Sept. 1993}} \times 100\%$$

$$= \frac{118.0}{114.7} \times \frac{139.4}{141.9} \times 100\%$$

$$\simeq 101.1\%.$$

Activity 42

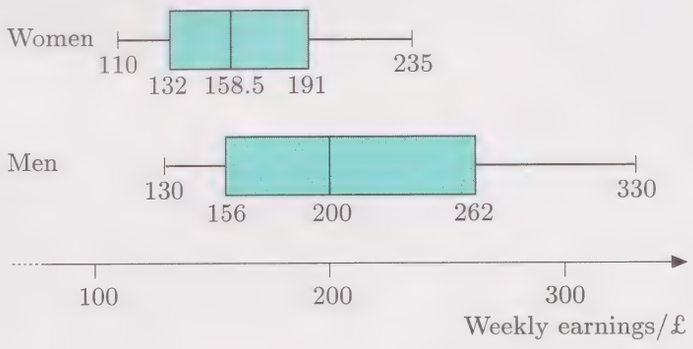
There are no comments on this activity.

Activity 43

There are no comments on this activity.

Activity 44

Moving out beyond the endpoints to suitable 'round' numbers suggests a scale with endpoints of £100 and £400 (say), so the scale must cover a range of £300. The actual scale will depend on the amount of space you have available. Here, because space is limited, a different scale has been used, and it has not gone out as far as £400. You are likely to have used a larger one. The same is true of the other boxplots in these comments. The boxplots for chefs' and cooks' earnings are shown below.

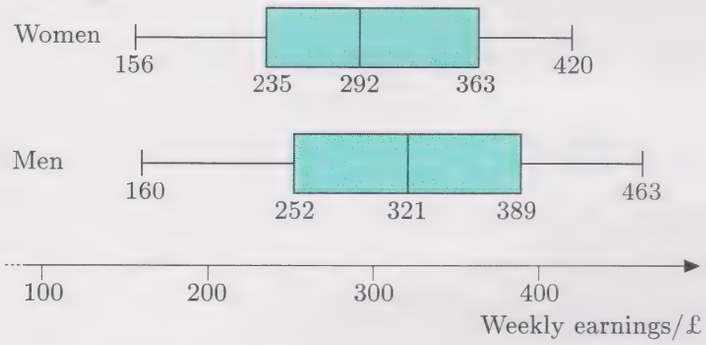


Boxplots of earnings data for chefs and cooks

Activity 45

(a) The sketches should look like the accurate diagram in part (b) (roughly!), but without the scale.

(b) In this case, possible 'round' numbers beyond the endpoints are £100 and £500, a range of £400. Taking £50 to correspond to 1 cm produces boxplots which fit comfortably on an A4 page; again, the higher figure £500 has been suppressed. Boxplots for the nurses' earnings are shown below.



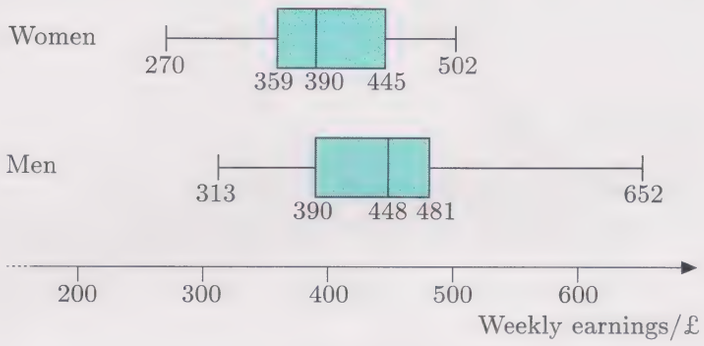
Boxplots of earnings data for nurses

(c) The earnings of the women are generally lower than the earnings of the men, but this difference is not great. All five values marked on the boxplots are slightly lower for the women.

Activity 46

(a) The sketches should look like the accurate diagram in part (b), but without the scale.

(b) Boxplots for the earnings of the secondary teachers are shown below.



Boxplots of earnings data for secondary teachers

(c) The earnings of the men were generally higher than those of the women. For example, the median earnings for the women is equal to the lower quartile of the men's earnings. All five values on the boxplots are lower for the women than for the men.

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Illustration

p. 24: Low Pay Unit.

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